Music 171: Additive Synthesis

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Adding Sinusoids at Different Frequencies

- Adding sinusoids of the **same** frequency produces another sinusoid at that frequency (with possibly different amplitude and/or phase).

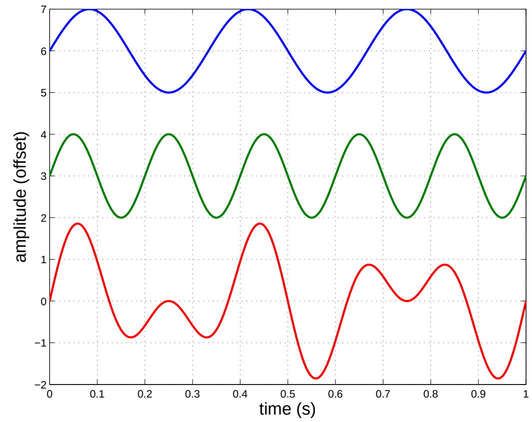


Figure 1: Adding 2 sinusoids at different frequencies.

- Adding sinusoids with **different** frequencies results in a signal that is no longer sinusoidal. But is it periodic?

A Non-Sinusoidal Periodic Signal

- If the frequencies of the added sinusoids are integer multiples of the fundamental, the resulting signal will be periodic.

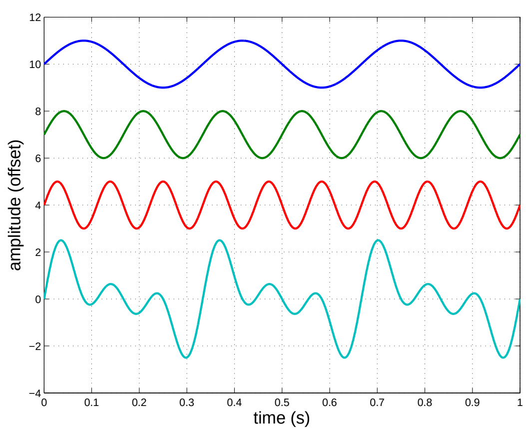


Figure 2: Adding sinusoids at 3, 6, 9 Hz produces a periodic signal at 3 Hz.

- Sinusoidal components, or *partials* that are integer multiples of a fundamental are calle *harmonics*.

Viewing in the Frequency Domain

- When viewing in the frequency domain, a “spike” at a frequency indicates there is a sinusoid in the signal at that frequency.

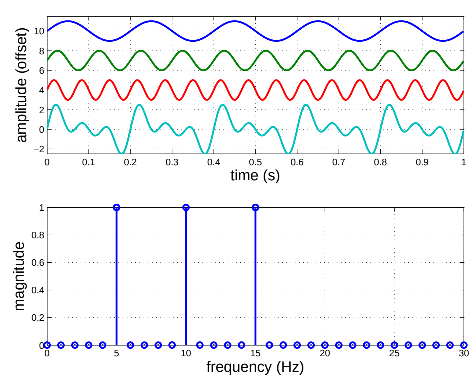
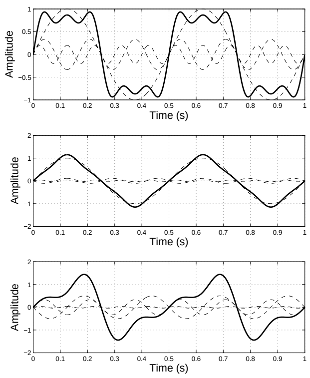


Figure 3: Adding sinusoids at 5, 10, 15 Hz in both time and frequency domain.

- The height of the spike here, indicates the amplitude (thus all sinusoids here have a peak amplitude of 1).
- The harmonic relationship between the partials can be seen by the even spacing between the “spikes”.

Periodic Waveforms

- A periodic waveform corresponds to a *harmonic spectrum*, one where sinusoidal components are *integer multiples* of a fundamental frequency.
- Periodic waveforms, or *harmonic* sounds, may be thought of as those having a pitch.
- "Standard" periodic waveforms: *square*, *triangle* and *sawtooth*, can be created using additive synthesis:



Creating Standard Periodic Waveforms

- The following table gives the *harmonic number*, relative *amplitude* and *phase* of the harmonics for standard waveforms.

Table 1: Other Simple Waveforms Synthesized by Adding Sinusoids

Type	**Harmonics**	**Amplitude**	**Phase (cosine)**	**Phase (sine)**
square	$n = [1, 3, 5, ..., N]$ (odd)	$1/n$	$-\pi/2$	0
triangle	$n = [1, 3, 5, ..., N]$ (odd)	$1/n^2$	0	$\pi/2$
sawtooth	$n = [1, 2, 3, ..., N]$ (even and odd)	$1/n$	$-\pi/2$	0

- Notice that the phase differs depending on whether you use a sine or cosine function.

Spectra of Standard Waveforms

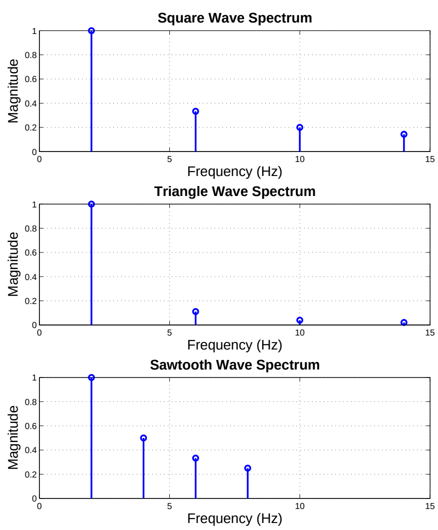


Figure 4: Spectra of complex waveforms

Harmonics and Pitch

- Notice that even though these new waveforms contain more than one frequency component, they are still periodic.
- Because the frequencis of components are **integer multiples** of some fundamental frequency, they are called *harmonics*.
- Signals with harmonic spectra have a fundamental frequency and therefore have a periodic waveform (the reverse is, of course, also true).
- **Pitch is our subjective response to the fundamental frequency**.
- The relative amplitudes of the harmonics contribute to the *timbre* of a sound, but do not necessarily alter the *pitch*.

Clarinet Analysis

- A clarinet is generally considered to be closed-open.
- Its tone (shown here in the steady state) can be viewed in both time and frequency domain.

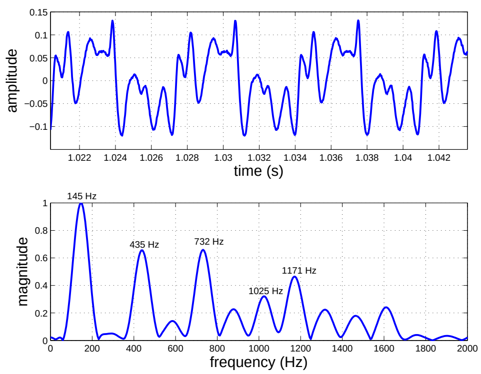


Figure 5: Frequency analysis of a clarinet note.

- Summing sinusoids at 145, 433 and 732 Hz (as well as others), can approximate a synthesis of the clarinet (in the steady state).

Additive Synthesis

- Discrete signals (bandlimited by half the sampling rate) may be represented as the sum of N sinusoids of arbitrary amplitudes, phases, AND frequencies:

$$x(t) = \sum_{k=0}^{N} A_k \cos(\omega_k t + \phi_k)$$

- We may therefore, synthesize a sound by setting up a bank of oscillators, each set to the appropriate amplitude, phase and frequency.
- The output of each oscillator is added to produce a synthesized sound, and thus the synthesis technique is called *additive synthesis*.

Additive Synthesis Pros and Cons

Pros

- Additive synthesis provides the maximum flexibility in the types of sound that can be synthesized.
- In certain cases, it can realize tones that are "indistinguishable from real tones."

Cons

- It is often necessary to do *signal analysis* before using additive synthesis to produce specific sounds.
- Often requires many oscillators to produce good quality sounds—computationally demanding!
- Attacks difficult.
- Many functions are useful only for a limited range of pitch and loudness, e.g.:
  - the timbre of a piano played at A4 is different from one played at A2;
  - the timbre of a trumpet played loudly is quite different from one played softly at the same pitch.

Modeling Transient Attacks

- It is possible to use some knowledge of acoustics to determine functions:
  - e.g.: higher frequency harmonics are often the last to appear in the attack but the first to decay.
  - implement using a separate envelope *on each harmonic* with a steeper attack/decay for lower frequencies.
- The duration of the attack and decay greatly influence the quality of a tone:
  - wind instruments tend to have long attacks, while
  - percussion instruments tend to have short attacks.

ADSR Envelope

- Recall the ADSR envelope (attack, decay, sustain, release) is an envelope that attempts to mimic the behaviour of sound produced by acoustic instruments.

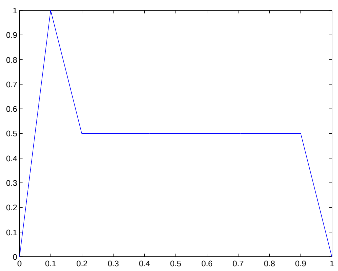


Figure 6: An ADSR envelope.

- Amplitude envelopes can occur on the overal sound or on individual sinusoidal components.

Sinusoid with an ADSR Envelope

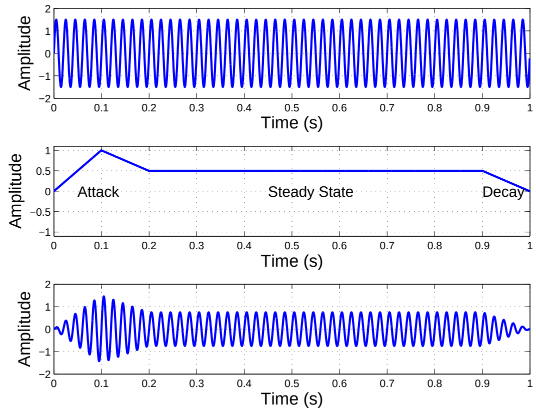


Figure 7: A sinusoid with an amplitude envelope.

Pitch and Frequency

- There is a nonlinear relationship between pitch perception and frequency.
  - e.g., the pitch interval of an octave corresponds to a frequency ratio of 2:1.
  - thus octaves in higher registers span more frequencies
- We will often encounter a pitch notation wich designates a pitch with an octave: C4 is middle C.
- The pitch A4 or "A440" is often used as a reference, and is an A at 440 Hz.
- What is the frequency one octave below A440?
- In equal-tempered tuning, there are 12 evenly spaced tones in an octave, called semi-tones:
  - The frequency n semitones above A440 is
    $$440 \times 2^{n/12} \text{ Hz.}$$
  - The frequency n semitones below A440 is
    $$440 \times 2^{-n/12} \text{ Hz.}$$