Music 171: Amplitude Modulation

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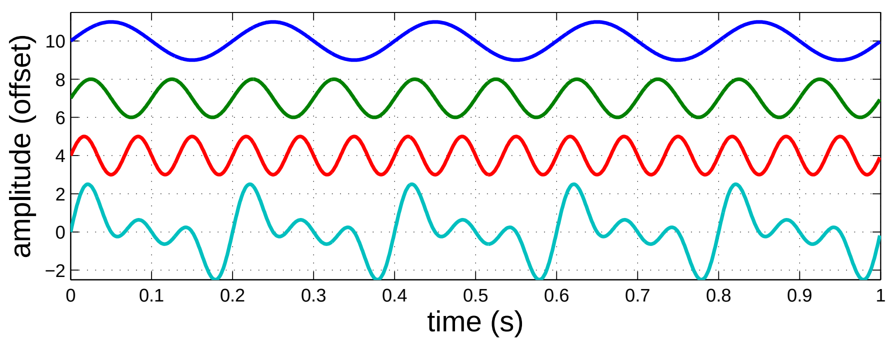
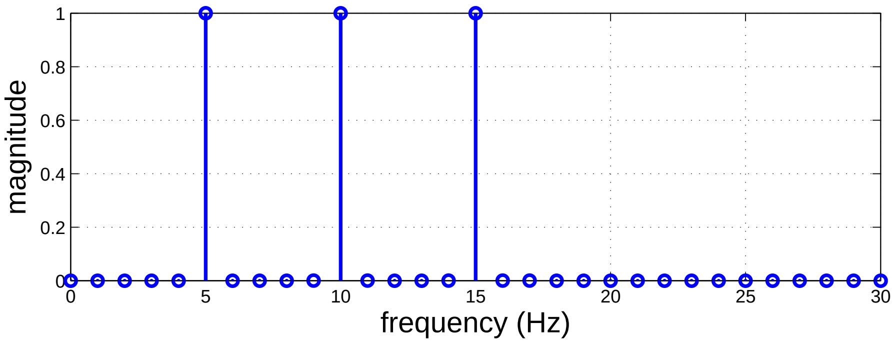
Adding Sinusoids

- Adding sinusoids having frequencies that are
  - **the same**, produces another sinusoid at that frequency;
  - **different**, produces a signal that is no longer sinusoidal;
  - **integer multiples of a fundamental frequency** f_0, produces a signal with period $1/f_0$.

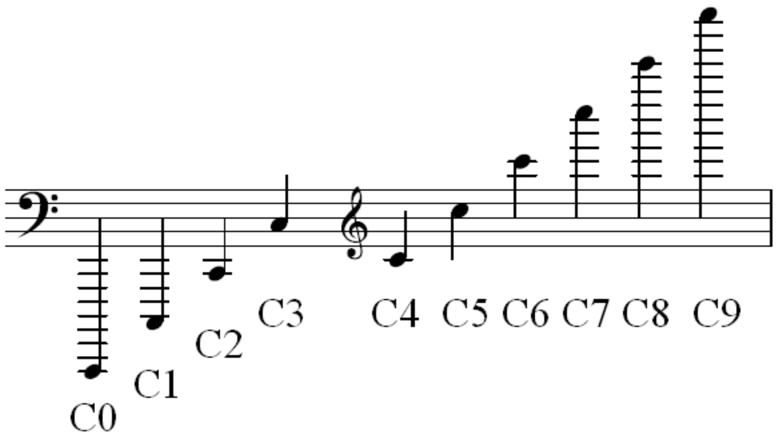




- Sinusoidal components that are integer multiples of a fundamental frequency are called *harmonics*.

A Note on Pitch and Frequency

- Generally, harmonic sounds are those for which we hear a pitch (you can hum them).
- **Scientific pitch notation** combines a note name with an octave number:
  - C4 is middle C;
  - A4 or A440 (440 Hz) is in the same octave (above) middle C and is often used as a reference tone.



- Recall, in equal-tempered tuning, there are 12 evenly spaced tones (semitones) in an octave,
  - The frequency n semitones above/below A440 is

$$440 \times 2^{\pm n/12} \text{ Hz}.$$

Summing Sinusoids Close in Frequency

- What happens when we two sinusoids having frequencies that are not harmonically related?
- Consider the sum of two sinusoids close in frequency:

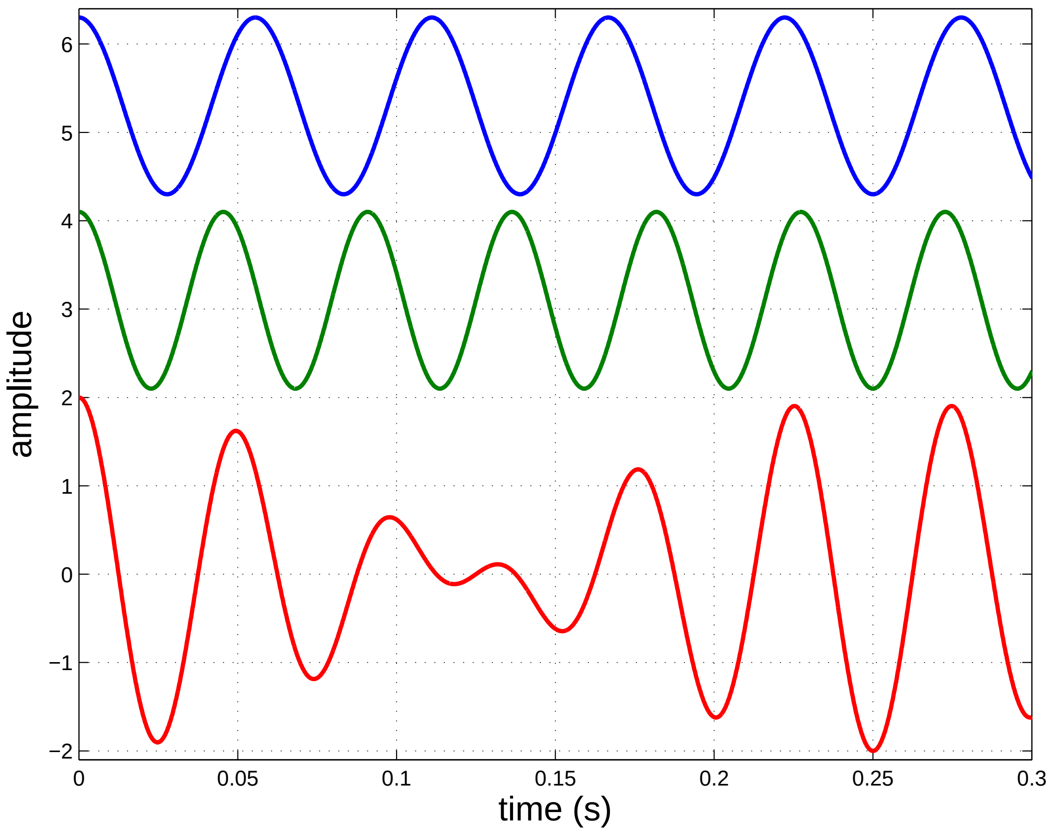


Figure 1: Sinusoids at 18 and 22 Hz.

An (non-linear) Amplitude Envelope

- Adding these two sinudoids produces a (seemingly) sinusoidal signal.
- But constructive/destructive interference imposes a (nonlinear) amplitude envelope.

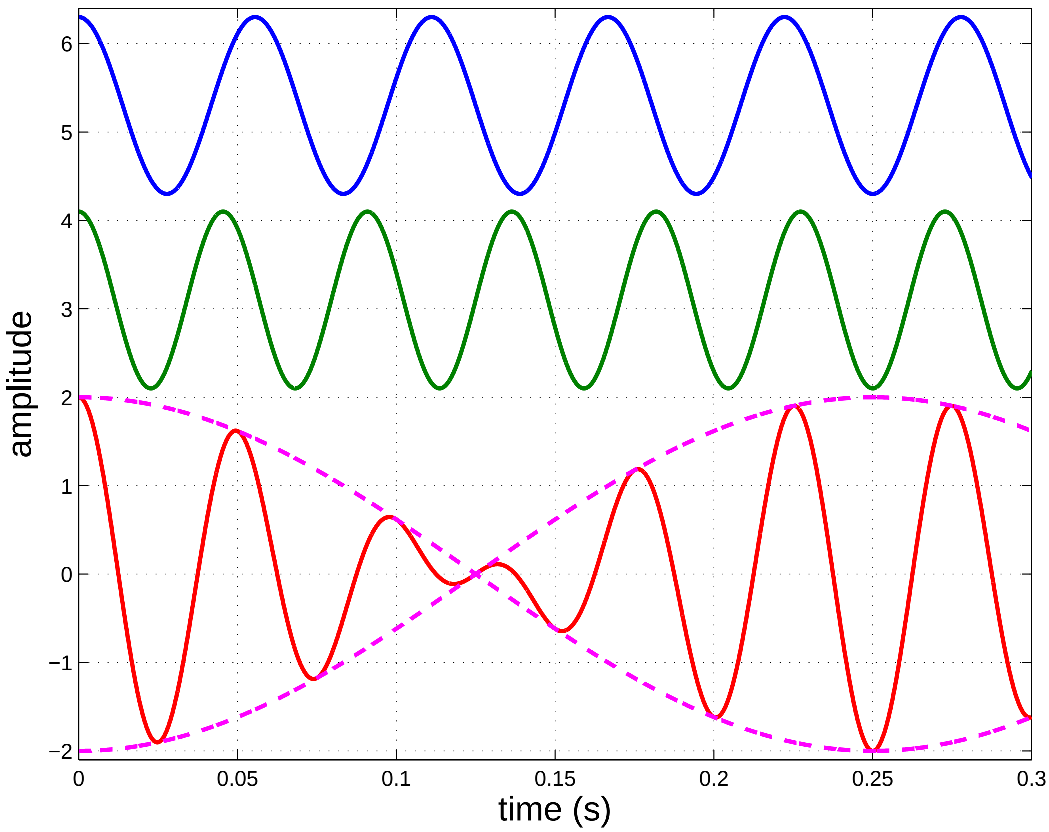


Figure 2: Sinusoids at 18 and 22 Hz.

Beat Notes

- Zooming out further, we see a sinusoid with a low-frequency sinusoidal amplitude envelope.

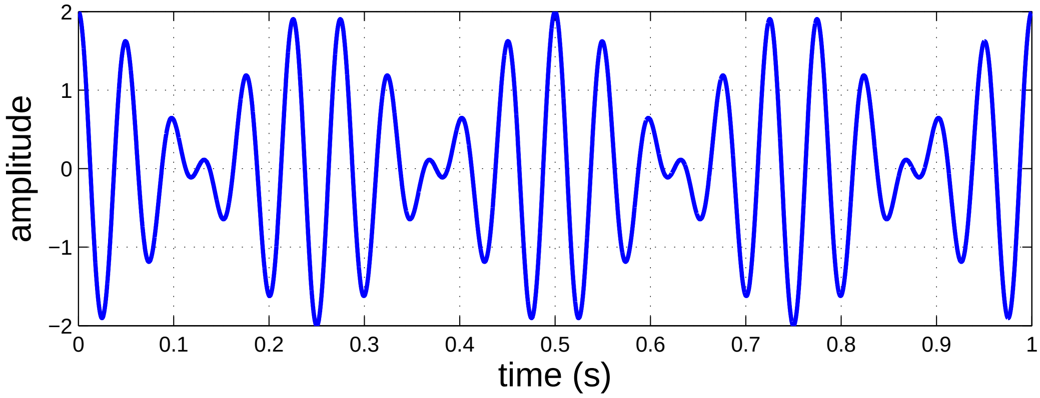


Figure 3: Sinusoids at 18 and 22 Hz.

- An audio version of this note (with frequencies at 218 and 222 Hz) can be heard here.
- The result is a sound that comes in and out of prominance, usually described as **beating**.
- Why does this effect occur?

Multiplication of Sinusoids

- To apply an envelope on a signal, the envelope is *multiplied* by the signal.
- The results then suggest that adding two sinusoids close in frequency **is the same** as multiplying two sinusoids, in this case, one low-frequency.
- This can be shown mathematically to be true!
- **Cosine Product** formula,

$$\cos(a)\cos(b) = \frac{\cos(a+b)+\cos(a-b)}{2},$$

we can show that

$$\begin{aligned} x(t) &= \cos(2\pi(220)t)\cos(2\pi(2)t) \\ &= \frac{1}{2}\left[\cos(2\pi(222)t)+\cos(2\pi(218)t)\right]. \end{aligned}$$

Beat Spectrum

- Sinusoidal multiplication can therefore be expressed as addition

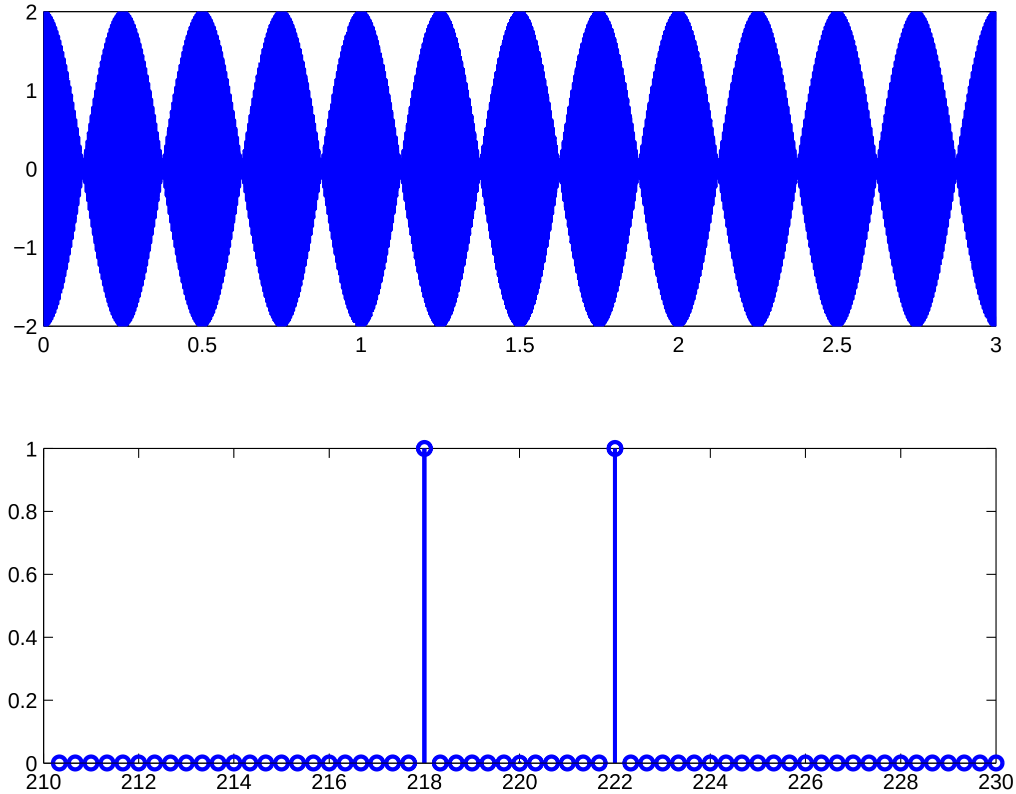


Figure 4: Beat note waveform and spectrum from adding sinusoids at 218 Hz and 222 Hz.

- Spectral frequencies are **not** those of the **multiplied** sinusoids (2 and 220 Hz), but their
  - **sum**: $220 + 2 = 222$ Hz and
  - **difference**: 220 - 2 $=$ 118 Hz.

Amplitude Modulation

- **Modulation** is the alteration of the amplitude, phase, or frequency of an oscillator by another signal.
- **Carrier**: the oscillator being modulated
- **Modulator**: the altering signal
- The spectral components generated by a modulated signal are called **sidebands** (or heterodynes).
- Three main techniques of amplitude modulation are:
  - Ring modulation
  - "Classical" amplitude modulation
  - Single-sideband modulation

  (we will not discuss the last one).

Ring Modulation

- Ring modulation (RM) (e.g. beat note): modulator is applied directly to the amplitude of the carrier:

$$x(t) = \cos(2\pi f_\Delta t)\cos(2\pi f_c t).$$

- Results in the sum of sinusoids:

$$x(t) = \frac{1}{2}\cos(2\pi(f_c - f_\Delta)t) + \frac{1}{2}\cos(2\pi(f_c + f_\Delta)t),$$

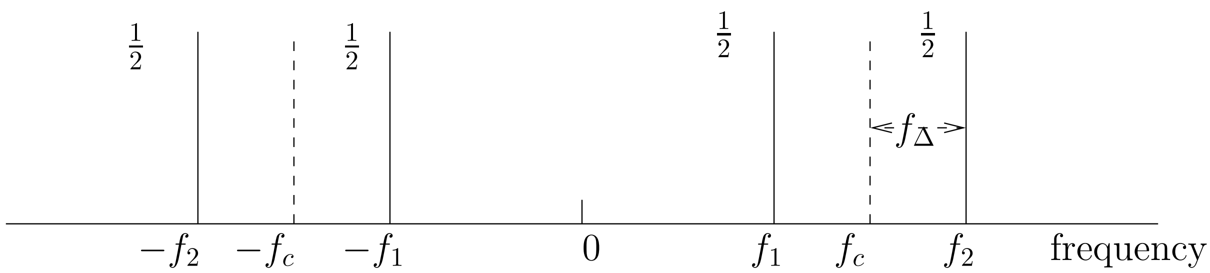


Figure 5: Spectrum of ring modulation.

- Neither carrier nor modulator are in the spectrum.
- Sometimes called double-sideband (DSB) modulation because of 2 produced sidebands.

Double Sideband Modulation

- RM can be realized by multiplying any two signals together (not just oscillators).
- Total number of frequency components:

$$N_{\textsf{total}} = 2 \times N_1 \times N_2$$

(2 times the product of the number of components in each signal).

“Classic” Amplitude Modulation

- Classic amplitude modulation (AM) is more general.
- Modulating signal includes a constant (DC offset):

$$x(t) = (A_0 + \cos(2\pi f_\Delta t))\cos(2\pi f_c t),$$

(where the first term is the modulating signal.)

- DC component $>= 1$ makes the modulating signal *unipolar*, i.e., the entire signal is greater than zero.

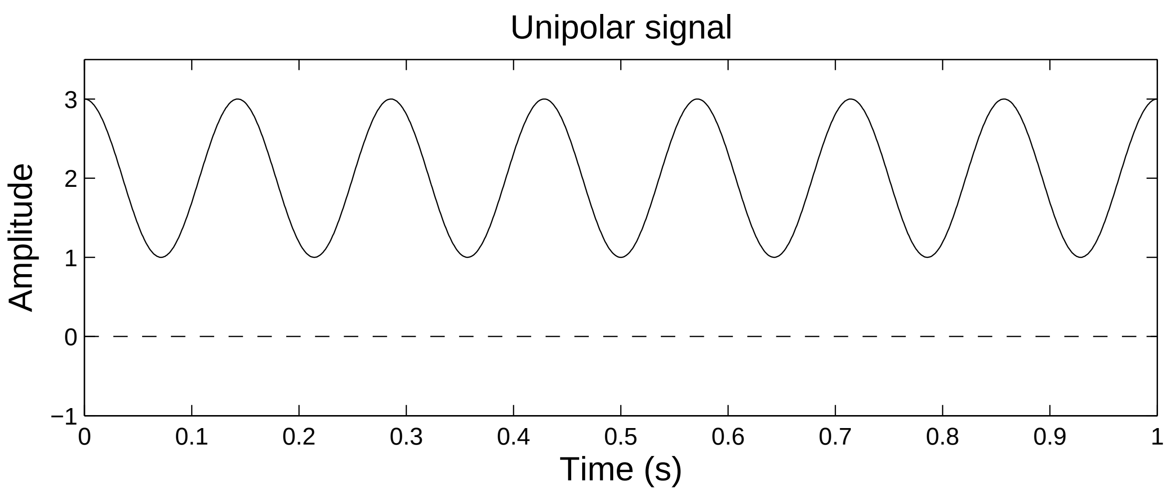


Figure 6: A unipolar signal.

Effects of the DC component

- Multiplying out the above equation, we obtain

$$x(t) = A_0 \cos(2\pi f_c t) + \cos(2\pi f_\Delta t)\cos(2\pi f_c t).$$

- The carrier frequency is now present in the spectrum.
- The second term can be expanded in the same way as was done for RM (i.e. the sidebands are identical).

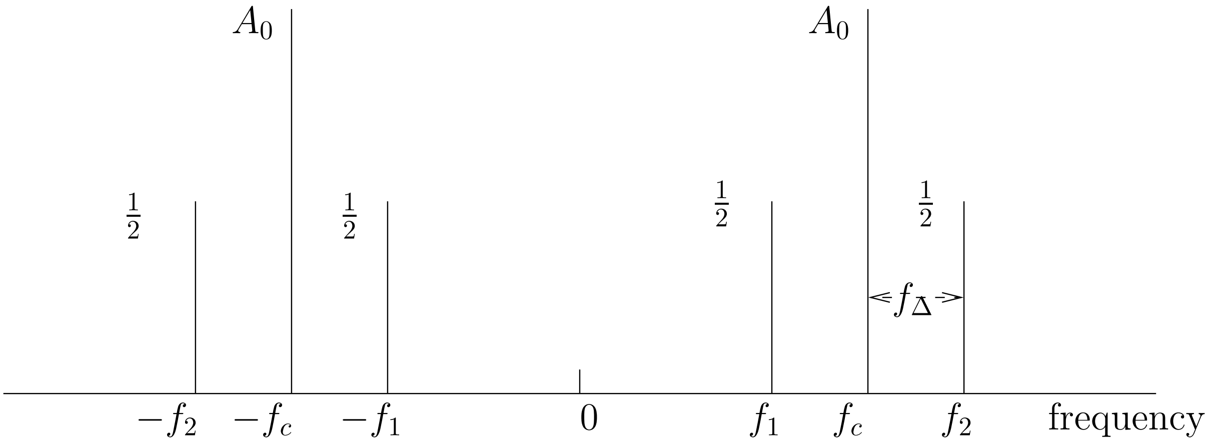


Figure 7: Spectrum of amplitude modulation.

RM and AM Spectra

- Sidebands are identical, but AM has center frequency f_c in the spectrum.

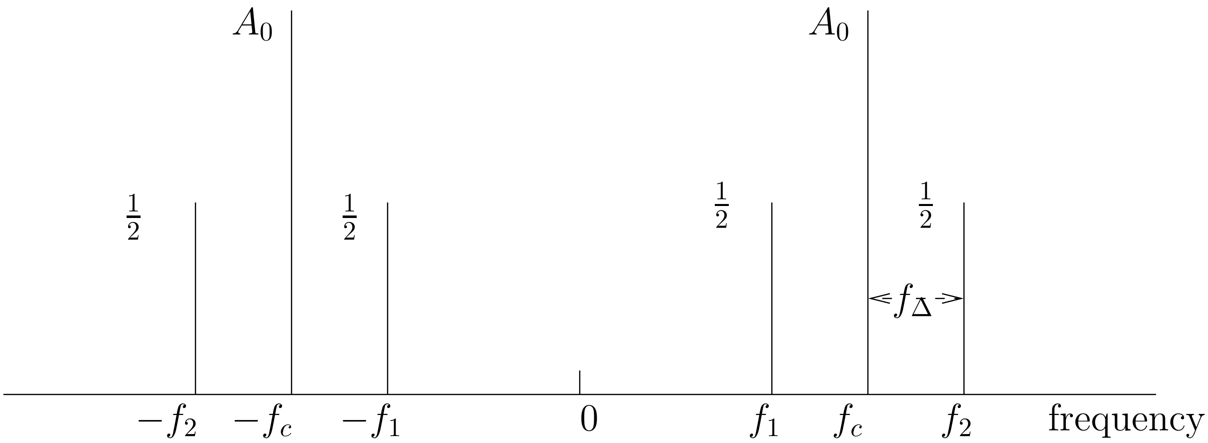


Figure 8: Spectrum of amplitude modulation.

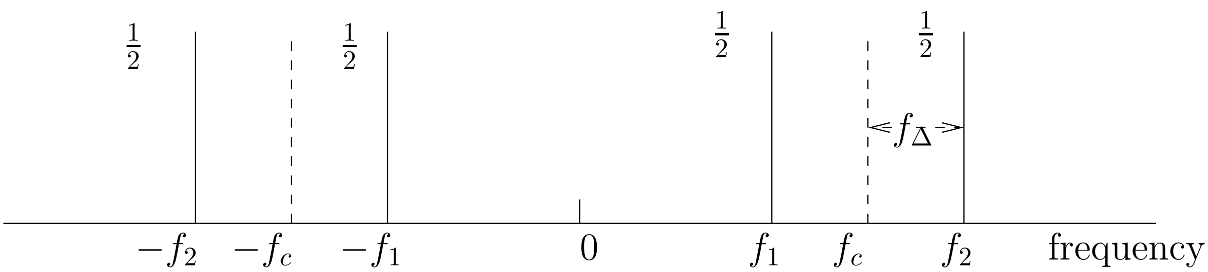


Figure 9: Spectrum of ring modulation.

- A DC offset in the modulator results in a spectrum with the carrier frequency f_c, at an amplitude equal to A_0.

RM and AM waveforms

- Waveforms for AM and RM showing effect of DC offset in the modulator:

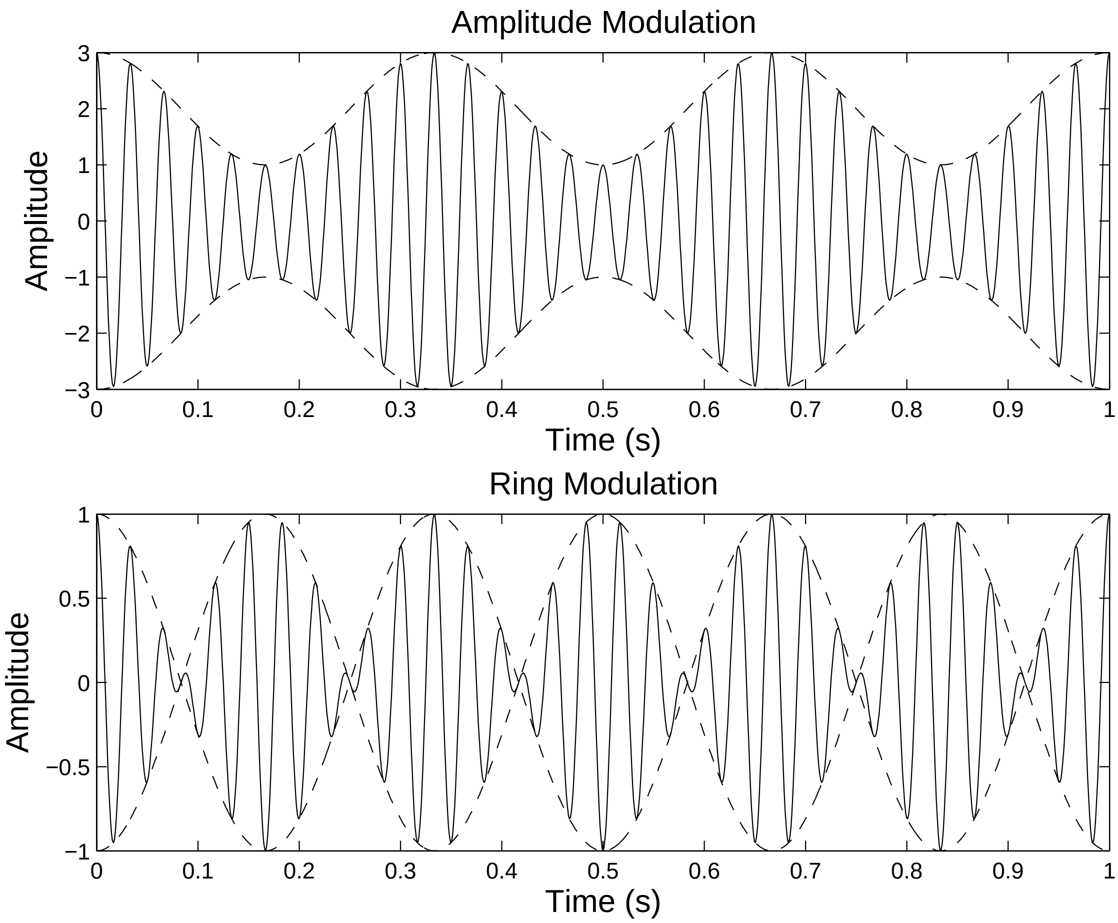
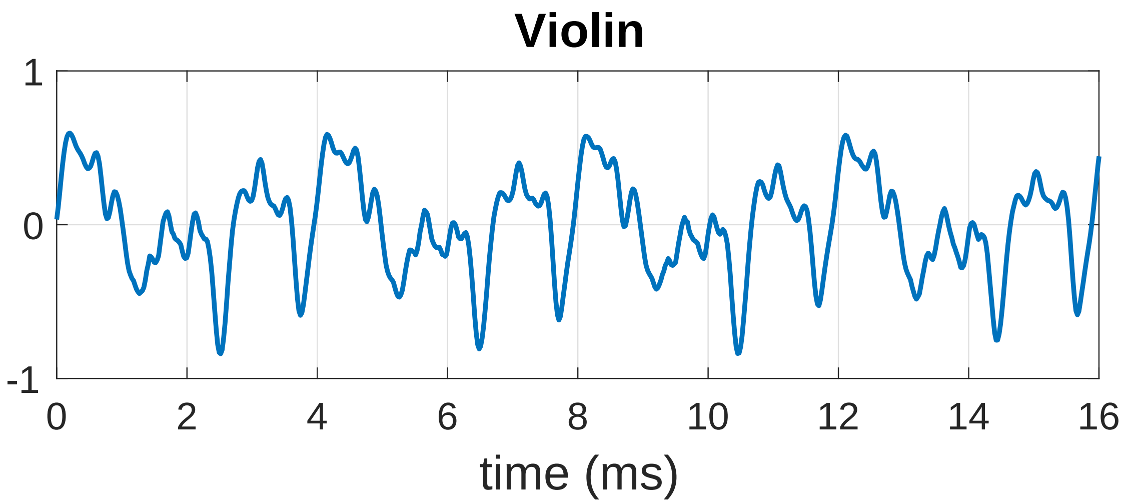
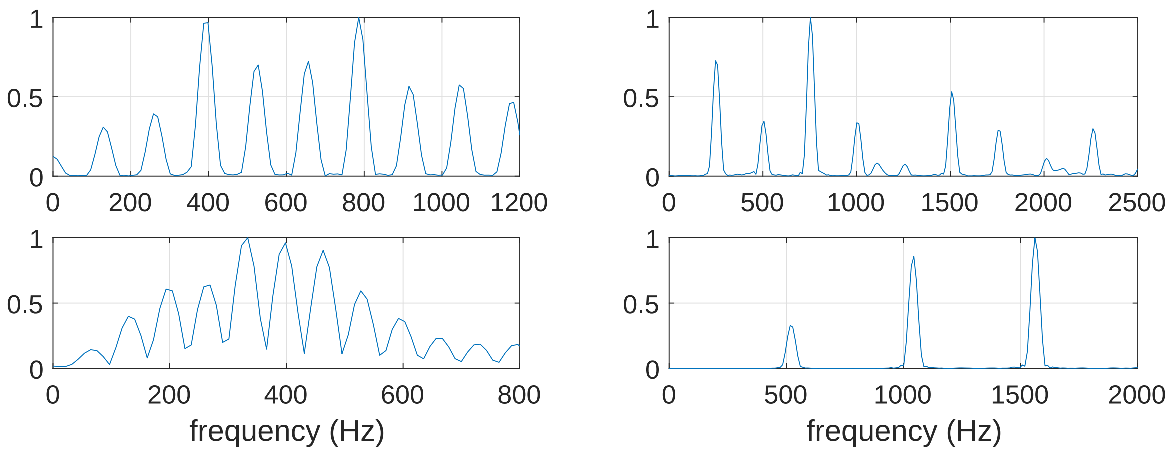


Figure 10: Amplitude and ring modulation.

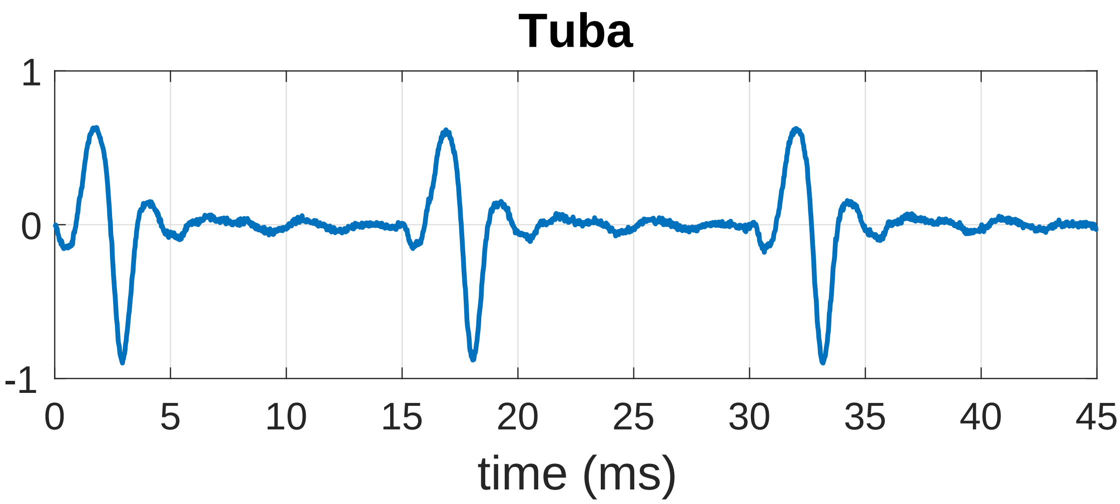
Review 1



- What is the Period?
- What is the Frequency?
- Which is the most likely spectrum?



Review 2



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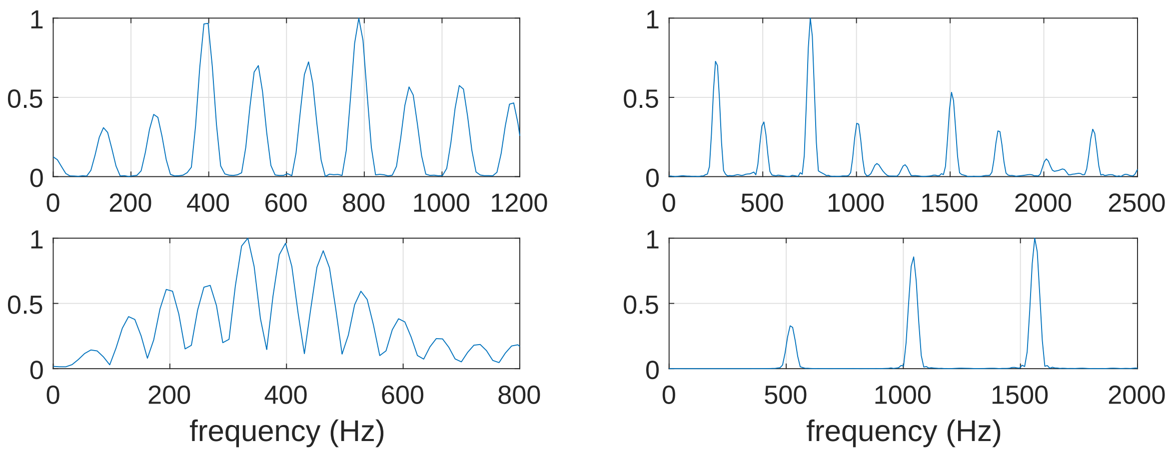
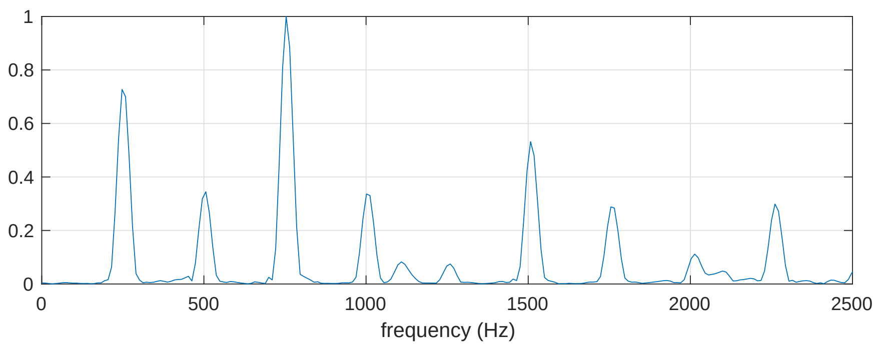


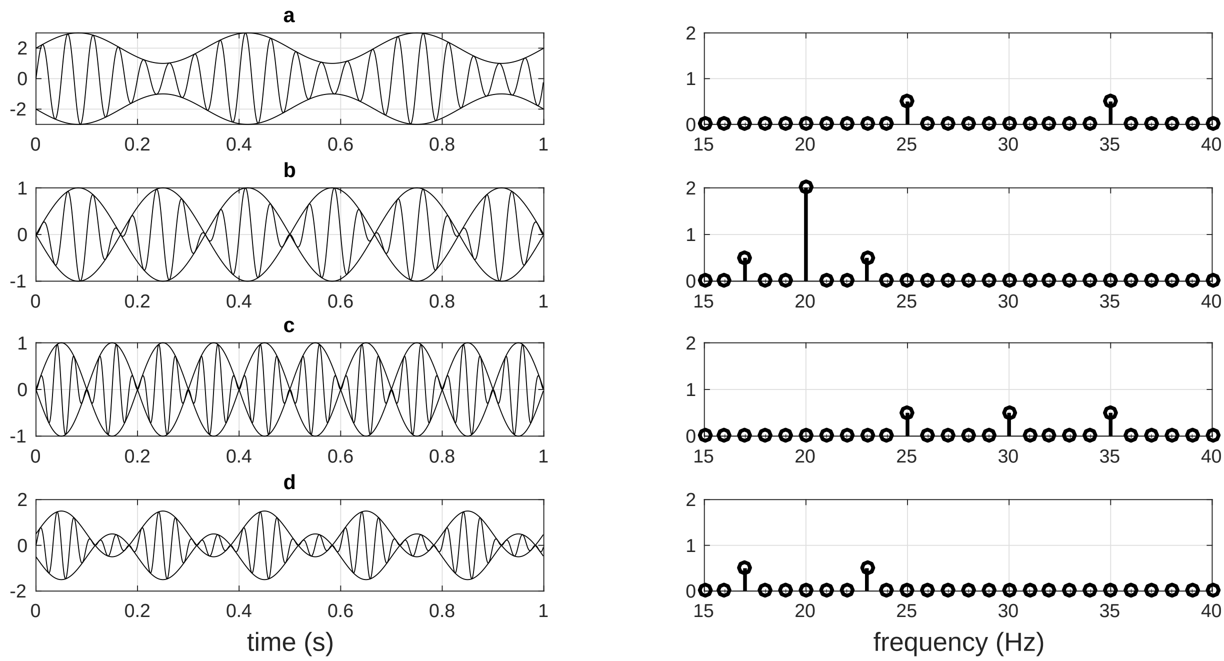
Review 4

- Provide frequencies, harmonic number and amplitude for components above .2 (for use in additive synthesis):



Review 4

- Match waveform and spectrum:



- Express as product and sum.