Music 270a: Digital Audio Processing
Assignment #1,
Due: Monday, October 14, 2019

Sinusoids and Sampling

1. (10 points) Define $x(t)$ as
$$x(t) = 4\sqrt{2}\sin(\omega_0 t + 45^\circ) + 3\cos(\omega_0 t).$$
Express $x(t)$ in the form
$$x(t) = A\cos(\omega_0 t + \phi),$$
where ϕ is in radians.

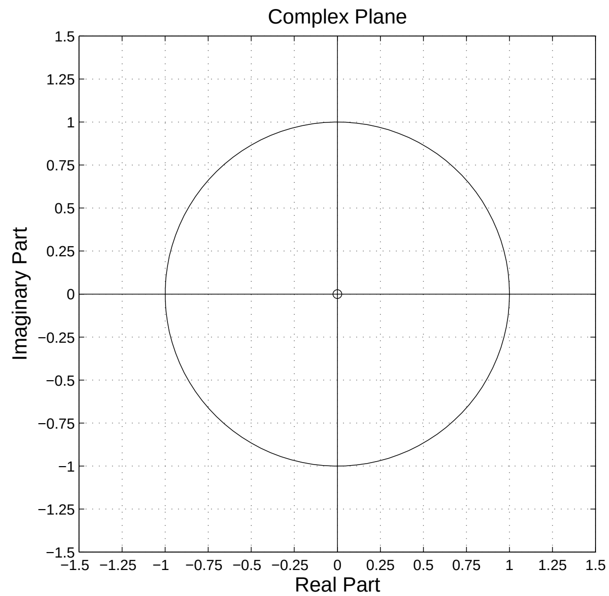
Hint(s):

- We know this can be expressed as a single sinusoid having the same frequency as the sinusoids in the sum since "adding sinusoids of the same frequency results in a sinusoid at that frequency").
- Express $\sin(\omega_0 t + 45^\circ)$ as a sum of sin and cos functions (using trigonometric identity: $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$). This will eliminate phase term.
- Group amplitudes for $\cos(\omega_0 t)$ and $\sin(\omega_0 t)$ terms and assign them variables B and C.
- Recall, a sinusoid can be expressed by the sum of an in-phase and phase-quadrature component (or zero-phase sine and zero-phase cosine):
$$A\cos(\omega t + \phi) = B\cos(\omega) + C\sin(\omega) \text{ where } A = \sqrt{B^2 + C^2} \text{ and } \phi = \tan^{-1}\left(\frac{-C}{B}\right)$$

2. (10 points) Given a sampling rate $f_s = 1/T_s$, where T_s is the sampling period, there is an infinite number of sinusoids, *aliases*, that will give the same sequence as $x(n) = A\cos(2\pi f_0 n T_s + \phi)$.
   (a) Sketch the magnitude of the spectrum of $x(n)$ for $f_0 = f_s/4$, over the range $-2f_s$ to $2f_s$. **Include all aliases for both negative and positive frequencies.**
   (b) On the same plot and over the same range, sketch the magnitude of the spectrum of $x(n)$ for $f_0 = 5f_s/8$. Use a broken line to distinguish this spectrum from that of the previous sinusoid.
   (c) If the input to a digital audio system is a sinusoid with a frequency $f_0 = 5f_s/8$ Hz , what will the output frequency be in terms of f_s?

Euler's formula

3. (10 points) From the inverse of Euler's formula, given by
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j},$$
we know that a real sinusoid can be represented in the complex plane as the scaled sum of two complex rotating vectors, one being the complex conjugate of the other.
   (a) Using the complex plane provided below, sketch and label the following phasors for $\phi = \pi/4$:
      i. the complex amplitude (phasor) $e^{j\phi}$;
      ii. its complex conjugate;



iii. the vector given by the sum of (i) and (ii);
iv. the vector given by the difference of (i) and (ii);

(b) Referring to the inverse of Euler's formulae above, write the algebraic expressions for the phasors obtained in (iii) and (iv) in terms of either cos or sin functions, and state whether they are real or imaginary.

(c) Using your above answers (and plots) as a guide, determine the relationship between $\sin(\phi)$ and $\cos(\phi)$ for $\phi = \pi/4$.

4. (10 points) Create a tone generator as a function in Matlab, having the interface:

```
function y = tonegen(f, dur, 'waveform', Nharm, fs)
%
% Y = TONEGEN(F, DUR, 'WAVEFORM', NHARM, FS) where
%   F is the fundamental frequency in Hz,
%   DUR is the duration in seconds,
%   'WAVEFORM' is either a sine, square, triangle or sawtooth wave,
%   NHARM is the number of harmonics, and
%   FS is the sampling rate.
```

Your function should create either a sine, square, triangle or sawtooth wave, with a specified fundamental frequency, duration, number of harmonics, and sampling rate.

- The square wave is composed only of odd-numbered harmonics with amplitudes in the ratio 1/n, where n is the harmonic number.
- The sawtooth wave has both odd- and even-numbered harmonics, with amplitudes in the ratio $1/n$.
- The triangle wave is composed only of odd-numbered harmonics with amplitudes in the ratio $(-1)^m/n^2$, where m is the index of harmonics present, and n is the harmonic number. That is, every other harmonic of the harmonics present is "out of phase".