Music 270a: Digital Audio Processing
Assignment #2
Due: Monday, October 21, 2019

Geometric Signal Theory and the DFT (15 points)

1. (5 points) Find the components (sample values) of the sinusoids $x_k(n) \triangleq e^{j\omega_k nT}$, where $\omega_k = 2\pi k f_s/N$, and $n = 0, 1, ..., N-1, k = 0, 1, ..., N-1$, and the dimension of the signal space is $N = 4$. For examples, $x_0 = [1, 1, 1, 1]$.

2. (5 points) Verify *orthogonality* of the set of signal vectors $\{x_k\}$ defined in the previous problem by computing the following *inner products*:

   (a) $\langle x_0, x_1 \rangle$,
   (b) $\langle x_0, x_2 \rangle$,
   (c) $\langle x_0, x_3 \rangle$,
   (d) $\langle x_1, x_2 \rangle$,
   (e) $\langle x_1, x_3 \rangle$,
   (f) $\langle x_2, x_3 \rangle$.

   Are these 6 combinations sufficient to show orthogonality?

3. (6 points) Recall the DFT is given by

$$X(\omega_k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j(2\pi kn/N)}, \quad k = 0, 1, 2, ..., N-1.$$

   Without using Matlab, find the length $N = 8$ DFT for the impulse

$$x(n) = [1, 0, 0, 0, 0, 0, 0, 0].$$

   Show your work. [Hint: take advantage of the fact that $x(n) = 0$ when $n \neq 0$.]

In Matlab...

(15 points)

4. (5 points) Create a function in Matlab called `plotspec`, having the following interface:

```
function y = plotspec(x, fs, 'option')
%
% PLOTSPEC Plot the magnitude of the fft.
%
%   Y = PLOTSPEC(X, FS, 'OPTION') plots the magnitude of the fft where
%   Y is the resulting magnitude, X is the input signal, FS is the
%   sampling rate, and 'OPTION' determines on which scale (linear or
%   dB) the magnitude should be plotted.
```

   Use your `plotspec` function to view a frequency representation of your audio signals, plotting the magnitude and phase responses in two subplots (using the `subplot` function).

5. (5 points) Create a Matlab function that returns the complex value signal, or analytic signal, of a real sinusoid $x(t)$ by using the Hilbert transform. Use your own Hilbert transform as described in class—Matlab's `hilbert` function returns the analytic signal rather than producing the phase-quadrature component.

6. (5 points)

   (a) Download the file `BbClar_ff_D3.wav` from the website. Read the file using Matlab's `audioread` function and then use your `plotspec` to plot the magnitude of the spectrum. At what frequencies are there significant peaks? Can you determine the fundamental frequency?

   (b) Use above observations to attempt an *additive synthesis* of the sound—i.e. one created by summing sinusoids having proper amplitudes and frequencies. Try using *amplitude envelopes* (on the whole sound as well as individual frequency components/sinusoids) to improve your synthesis.