Music 270a: Digital Audio Processing
Assignment #5 (optional)
Due: Monday, December 2, 2019

1. Recall that the gain of the simple first-order FIR low-pass filter ($y(n) = x(n) + x(n-1)$) is given by:

$$\begin{aligned} G(\omega) &= |H(\omega)| \\ &= |1 + e^{-j\omega T}| \\ &= |e^{-j\omega T/2}(e^{j\omega T/2} + e^{-j\omega T/2})| \\ &= \left|2\cos(\omega T/2)e^{-j\omega T/2}\right| \\ &= 2\cos(\omega T/2). \end{aligned}$$

   Provide an expression for the cutoff frequency f_c, i.e. the frequency at which the power at DC is reduced by 1/2, or equivalently, the amplitude (pressure) is reduced by $\sqrt{1/2}$.

2. Derive an expression for the gain of the filter produced by the cascade of two (2) simple low-pass FIR filters each given given by $y(n) = x(n) + x(n-1)$. As in question 1, provide an expression for its cutoff frequency f_c.

3. Derive an expression for the gain of the filter produced by the cascade of three (3) simple first-order low-pass FIR filters. How does the result differ from the "pattern" established by the first- and second-order frequency responses?

4. Write a function (`cutoff.m`) that determines the normalized cutoff frequency given feedforward (B) and feedback (A) coefficients. Your function should have the following interface:

```
function fc = cutoff(B,A)
```

   and will likely make use of Matlab's `freqz` function for obtaining the filter's frequency response. You can use the `find` function along with a logical expression to look for particular values within the frequency response.

5. Call your `cutoff` function by setting B coefficients to the impulse response of a cascade of seven (7) simple low-pass filters and determine the normalized ($f_s = 1$) cutoff frequency rounded to 1 decimal place. Include your answer with your writeup.

6. Recall that linear-phase filters have a symmetric impulse response, i.e.

$$h(n) = h(N - 1 - n),$$

   for an impulse response of length N. A symmetric impulse response thus corresponds to a real frequency response times a linear phase term $e^{-j\alpha\omega T}$, where α is the slope of the phase. A zero-phase filter is a special case of a linear-phase filter in which the phase slope is $\alpha = 0$, and the impulse response is symmetric about 0 and thus even

$$h(n) = h(n - 1).$$

   Note this means zeros phase filters are not causal.

   (a) If the spectrum is real, what are the two possible values for phase?

(b) Implement the following low-pass filter using Matlab's `firpm` function and using the following parameter values:

```
N = 11;                 % filter order
b = [0 .1 .2 .5]*2];  % band edges
M = [1 1 0 0];          % amplitudes at b
h = firpm(N-1, b, M); % impulse response
```

(c) Plot impulse response using `stem`. You should see it is symmetric and thus in linear form.

(d) Convert to zero phase by left (circular) shifting $(N-1)/2 = 5$ and making it symmetric about zero.

(e) Take the FFT of both linear and zero phase impulse responses. What do you notice? How do their amplitude and phase responses compare (refer back to question (a))?