Music 171: Frequency Modulation

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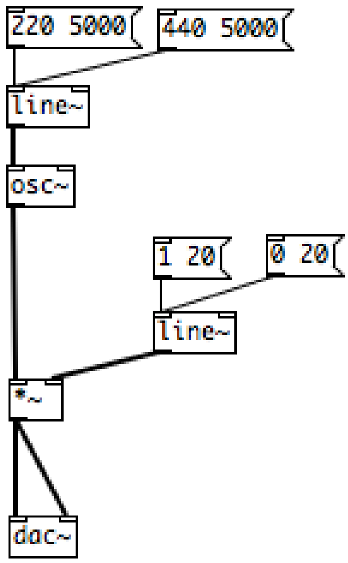
Changing Frequency

- We have already seen (in Assignment 1) how we can linearly change the frequency of a sinusoid using the **line**$^\sim$ object.



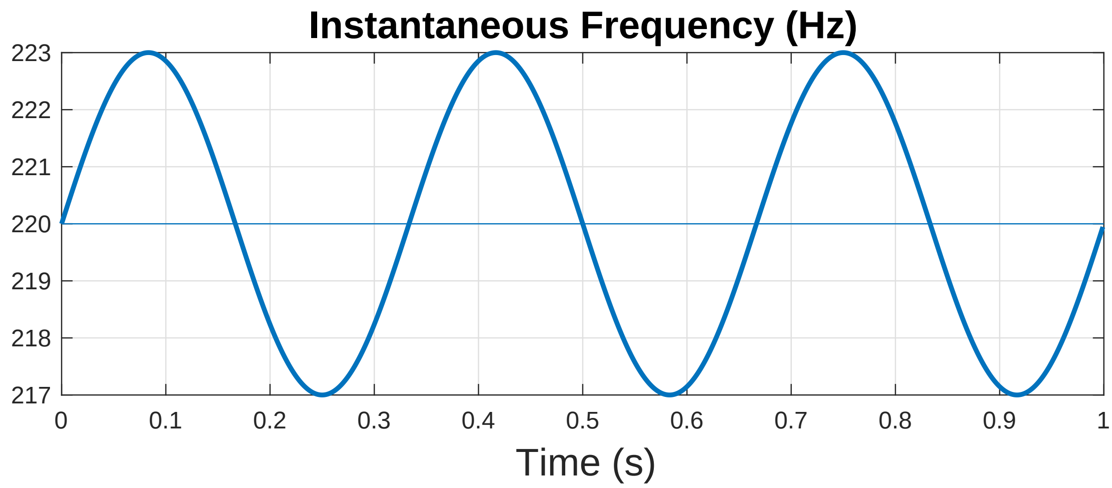
- This signal is generally called a *chirp*, as it sweeps from one frequency
- Logarithmic chirps, as well as other *continuous* changes in frequency are also possible.

Vibrato

- **Vibrato**: a wavering of pitch.
  - occurs in the singing voice and musical instruments (e.g. violin, wind instruments, the theremin, etc.)
  - often due to human control *after* note onset;
  - center frequency f_c is modulated by a sinusoid with frequency f_m yielding `osc~` frequency:

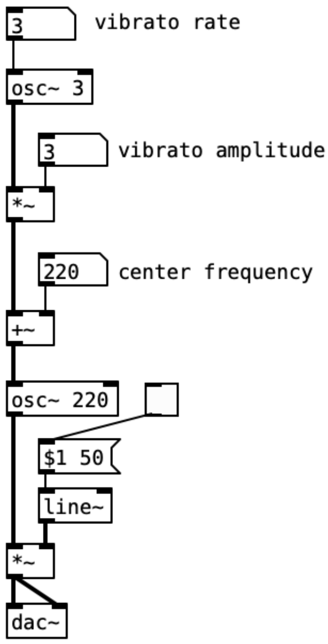
$$f_i = f_c + d \sin(2\pi f_m t),$$

    where d is the frequency deviation from f_c.
  - example: $f_c = 220$ Hz, $f_m = 3$ Hz, and $d = 3$:



Vibrato Simulation

- As a sinusoid `osc~` modulated the *amplitude* of a carrier in AM/RM, so can it modulate the *frequency* of a carrier to implement FM vibrato:



- The *width* of the vibrato (the deviation from the carrier frequency) determined by amplitude d.
- The *rate* of the vibrato, determined by f_m.
- To be perceived as vibrato, rate **must be below the audible frequency range** (LFO) and the width (amplitude) made small.

FM Synthesis of Musical Instruments

- When the vibrato rate is increased to a frequency in the audio range, and when the width is made larger, the technique can be used to create a broad range of distinctive timbres.
- Frequency modulation (FM) synthesis was invented by John Chowning at Stanford University's *Center for Computer Research in Music and Acoustics* (CCRMA).
- FM synthesis uses fewer oscillators than either additive or AM synthesis to introduce more frequency components in the spectrum.

FM Sidebands

- The upper and lower sidebands produced by FM are grouped in pairs according to the harmonic number of f_m, that is, frequencies present are given by $f_c \pm k f_m$.

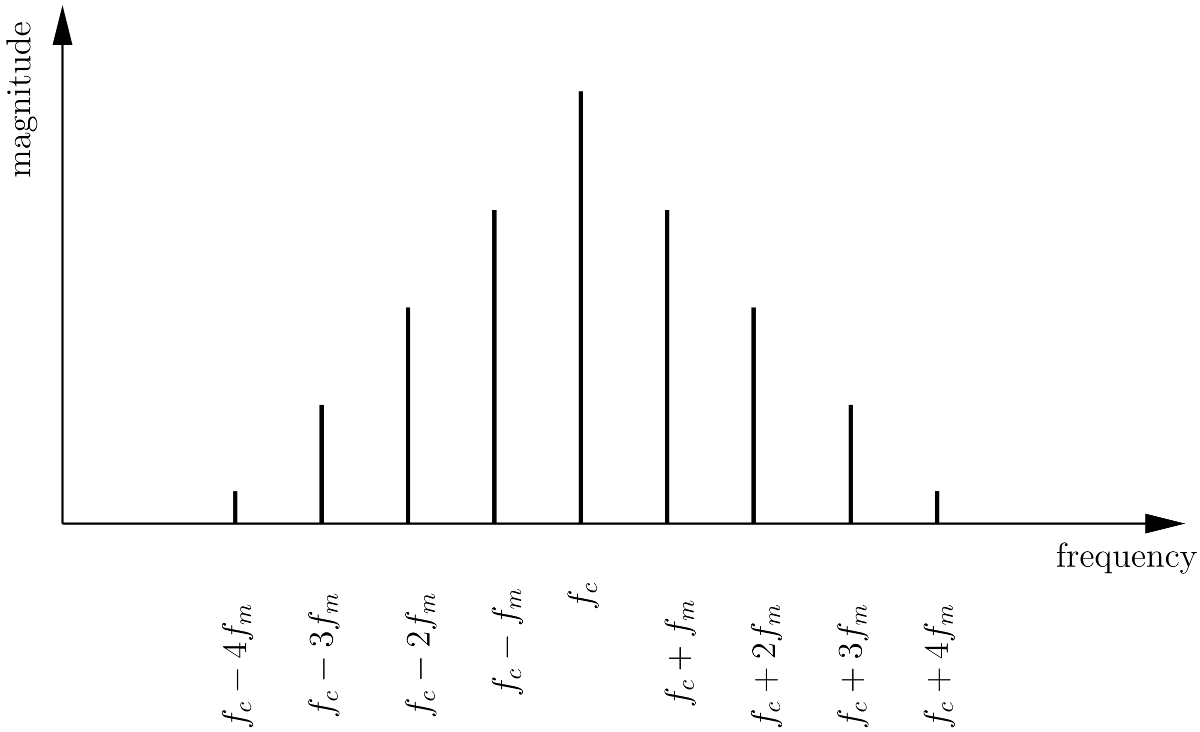


Figure 1: Sidebands produced by FM synthesis.

FM Bandwidth

- The deviation d (amplitude of the modulator) acts as a control of FM bandwidth and is usually set as the product of f_m and the **modulation index** I:

$$d = I f_m$$

so that instantaneous frequency is

$$f_i = f_c + I f_m \sin(2\pi f_m t).$$

- Modulation index I determines **harmonic content**:
  - in general, the highest sideband that has significant amplitude is *approximately*

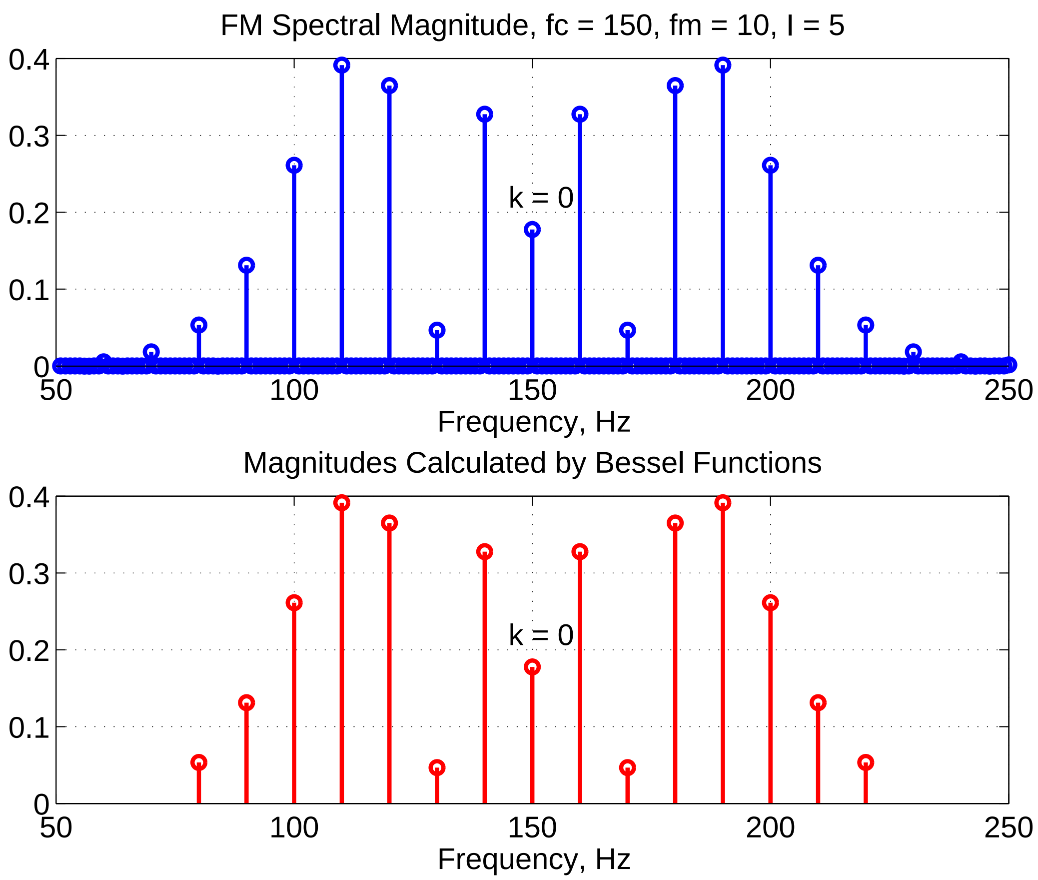
$$k_{\text{max}} = I + 1.$$

- If $I(t)$ is time varying, the timbre of a tone can change over time (as do musical sounds!).
- Though calculating the actual amplitude of sidebands is beyond the scope here, it is interesting to known they can be described by **Bessel functions**.

FM Spectrum—Actual vs. Bessel

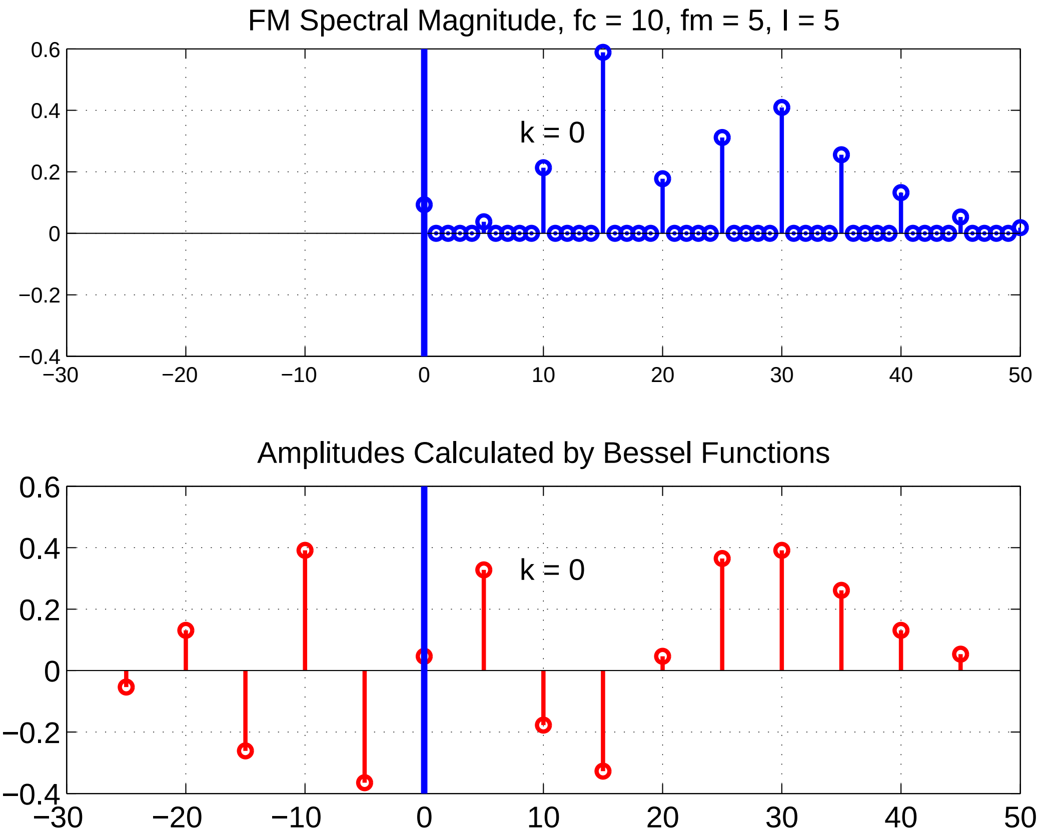
- The following shows the spectrum and sideband amplitudes calculated using Bessel functions for FM parameters:

$$f_c = 150, \quad f_m = 10, \quad I = 5.$$



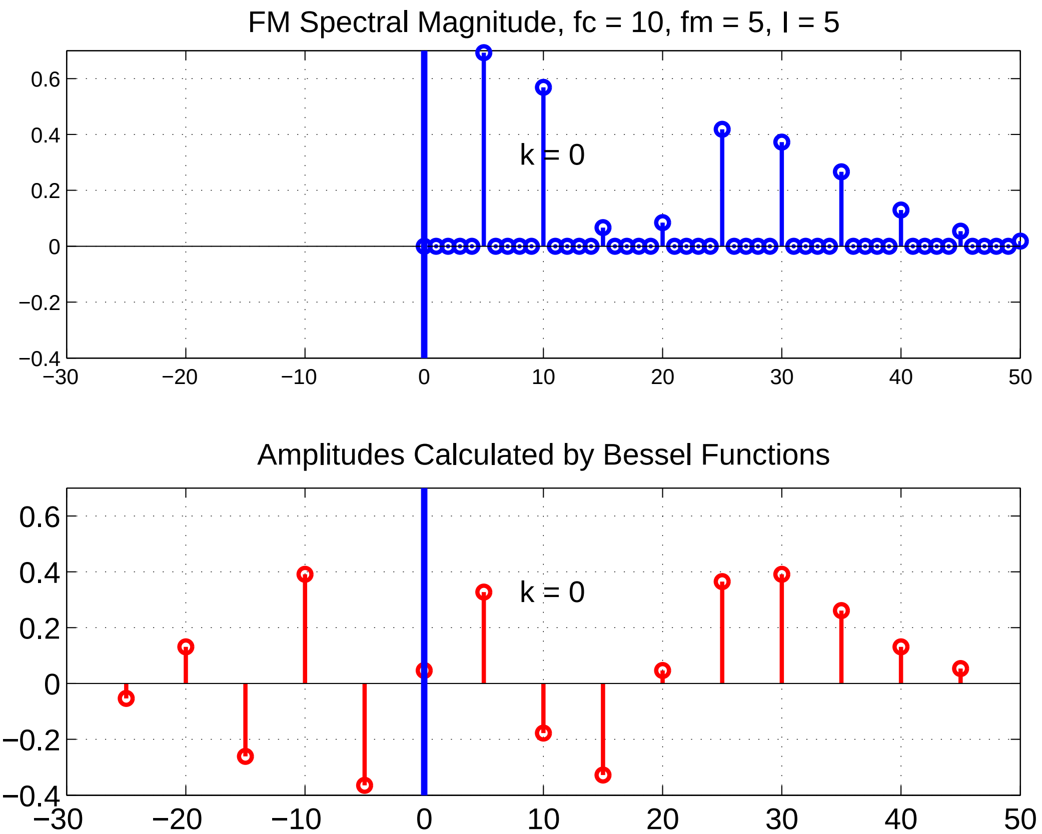
Effect of Phase in FM

- If the FM spectrum spreads downward below 0 Hz, "negative" components are folded over the 0 Hz axis to their corresponding positive frequencies.



Changing Carrier Phase

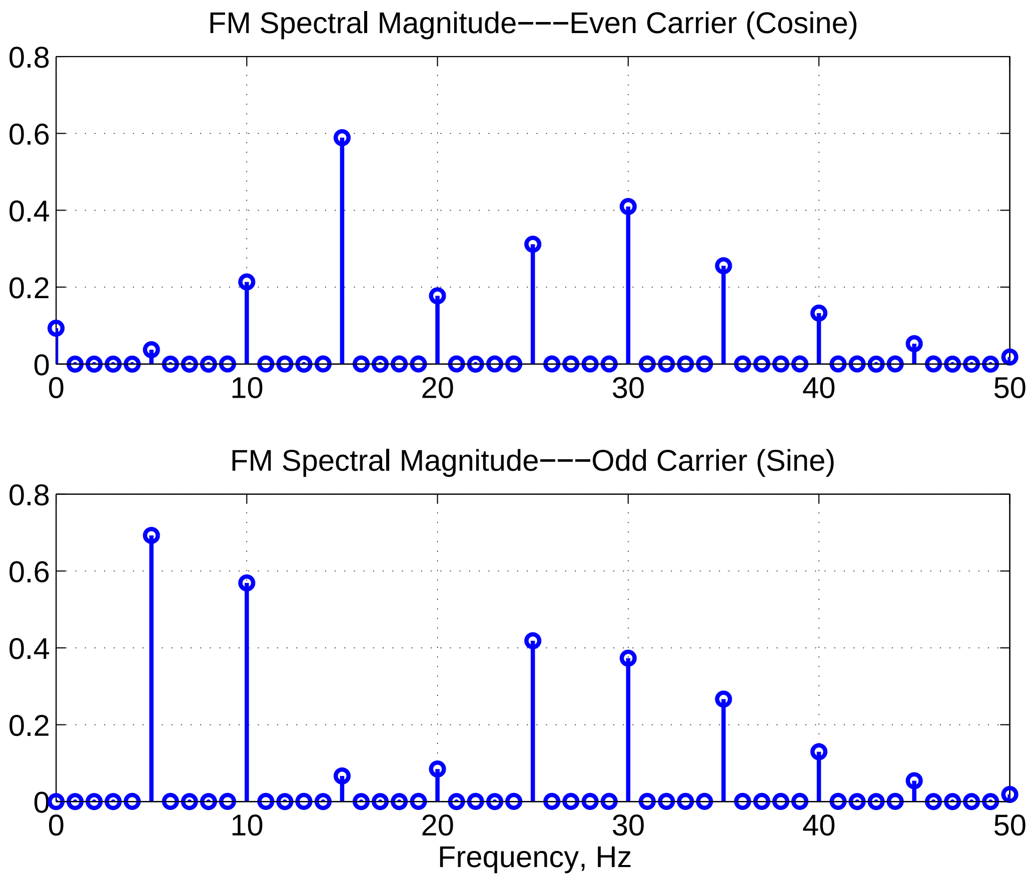
- Furthermore, if the carrier is an odd function, the act of folding reverses the phase.



- Notice the cancellation at DC.

Carrier as Even Odd Functions

- Notice how the difference in carrier phase is concentrated in the low-frequency region where overlap from the 0 Hz axis causes constructive/destructive interference.



Fundamental Frequency in FM

- The fundamental (or sounding) frequency of an FM sound, is found by first representing the ratio of the carrier and modulator frequencies in reduced form:

$$\frac{f_c}{f_m} = \frac{N_1}{N_2}$$

(N_1 and N_2 are integers with no common factors).

- The fundamental frequency is then given by

$$f_0 = \frac{f_c}{N_1} = \frac{f_m}{N_2}.$$

- Example: carrier frequency $f_c = 10$ and modulator frequency $f_m = 5$ yields the ratio in reduced form:

$$\frac{f_c}{f_m} = \frac{10}{5} = \frac{2}{1} = \frac{N_1}{N_2},$$

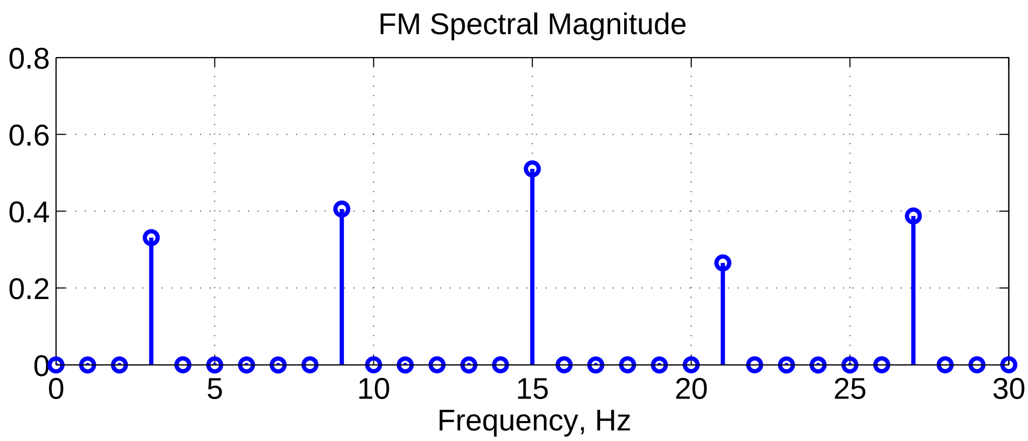
and a fundamental frequency of

$$f_0 = \frac{f_c}{N_1} = \frac{10}{2} = \frac{f_m}{N_2} = \frac{5}{1} = 5.$$

A “Different” Fundamental

- Consider a case where the fundamental is not equal to either carrier or modulator frequency:
- The ratio of $f_c = 9$ to $f_m = 6$ is 3:2 and the fundamental frequency is given by

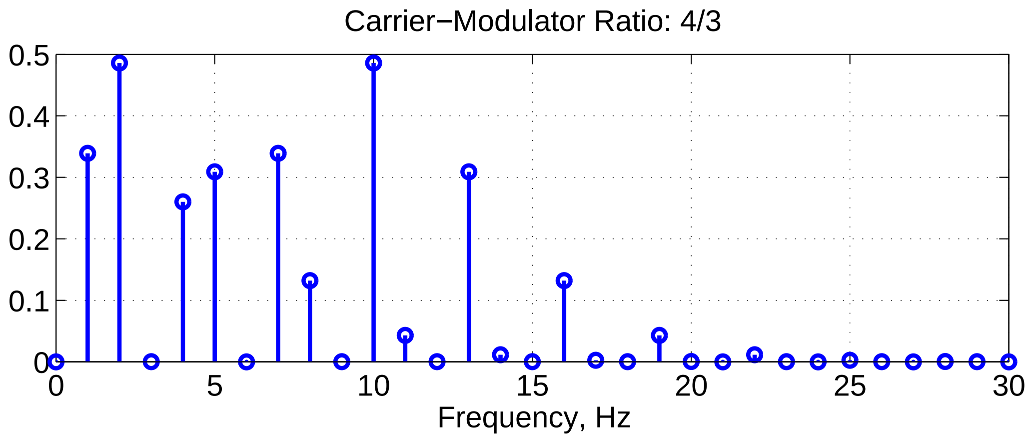
$$f_0 = \frac{9}{3} = \frac{6}{2} = 3.$$



- Notic the spectrum has only odd harmonics!

Missing Harmonics in FM

- If $N_2 > 1$, every N_2^{th} harmonic is missing.
- E.g: ratio of carrier to modulator of 4:3 for $N_2 = 3$:



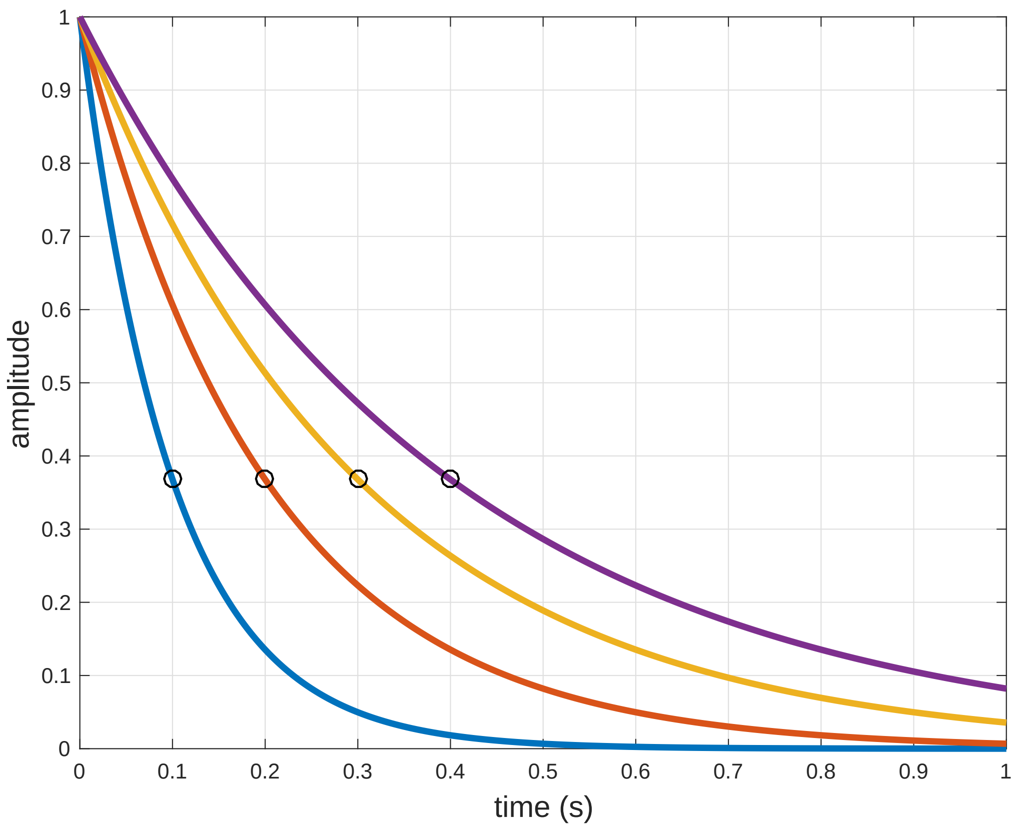
- Notice the fundamental frequency f_0 is 1, but every third multiple of f_0 is missing from the spectrum.

Exponential Envelope

- The exponential (decay) envelope is given by

$$e^{-t/\tau}$$

- τ ("tau"): time to decay to $1/e$ (≈ 0.368).



Note duration ($T60$)

- **Note duration**: time to decay 60 dB ($T60$), or to 0.001[1] (linear scale), the point of inaudibility.
- At $t = T60$, the amplitude is

$$e^{-T60/\tau} = 0.001.$$

- Taking the $\log_e$ of both sides yields

$$-T60/\tau = \log_e(0.001).$$

- Given a value for τ, the note duration is

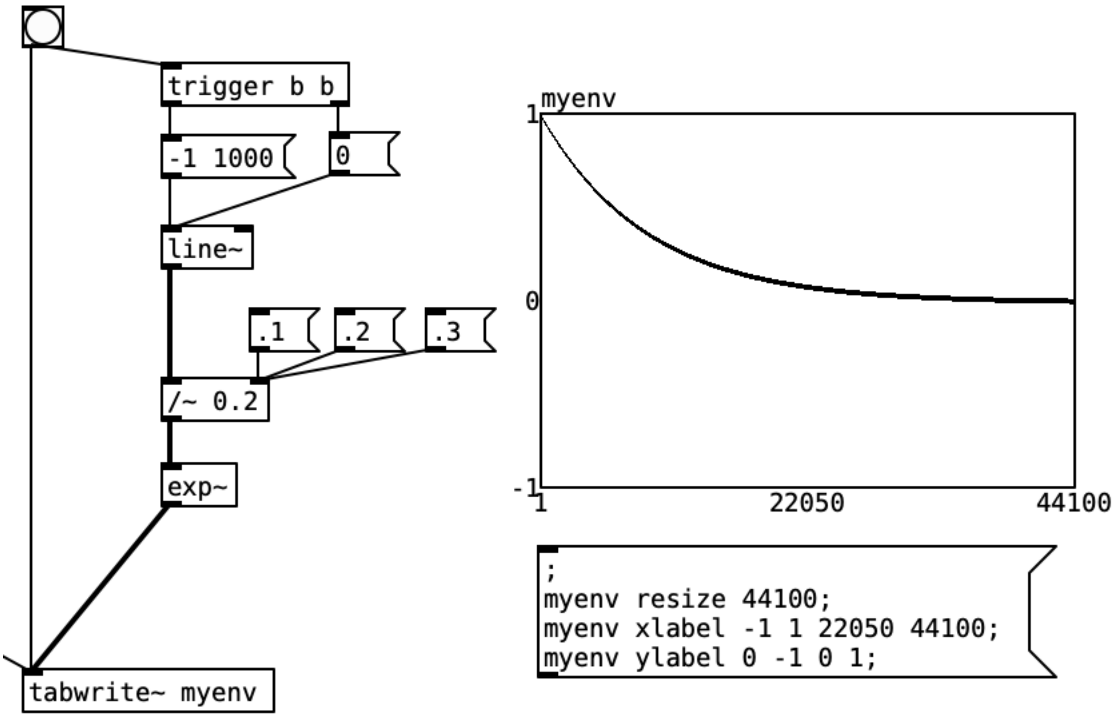
$$T60 = -\tau \log(0.001).$$

- Given note duration $T60$: the characteristic time constant is

$$\tau = -T60/\log(0.001).$$

[1] If $20\log_{10}(a) = 60$ dB, then $\log_{10}(a) = 60/20$, and taking each side to a power of 10 yields $a = 10^3 = .001$.

Exponential Envelope in Pd



Some FM instrument examples

- When implementing simple FM instruments, we have these main parameters that will effect the overall sound:
  1. The carrier and modulator frequency and phase
  2. The maximum modulating index
  3. The envelopes that define how the amplitude and modulating index evolve over time.
- Using the information taken from John Chowning's article on FM we may develope envelopes for the following simple FM instruments:
  - bell-like tones,
  - wood-drum
  - brass-like tones
  - clarinet-like tones

FM Instrument Envelopes

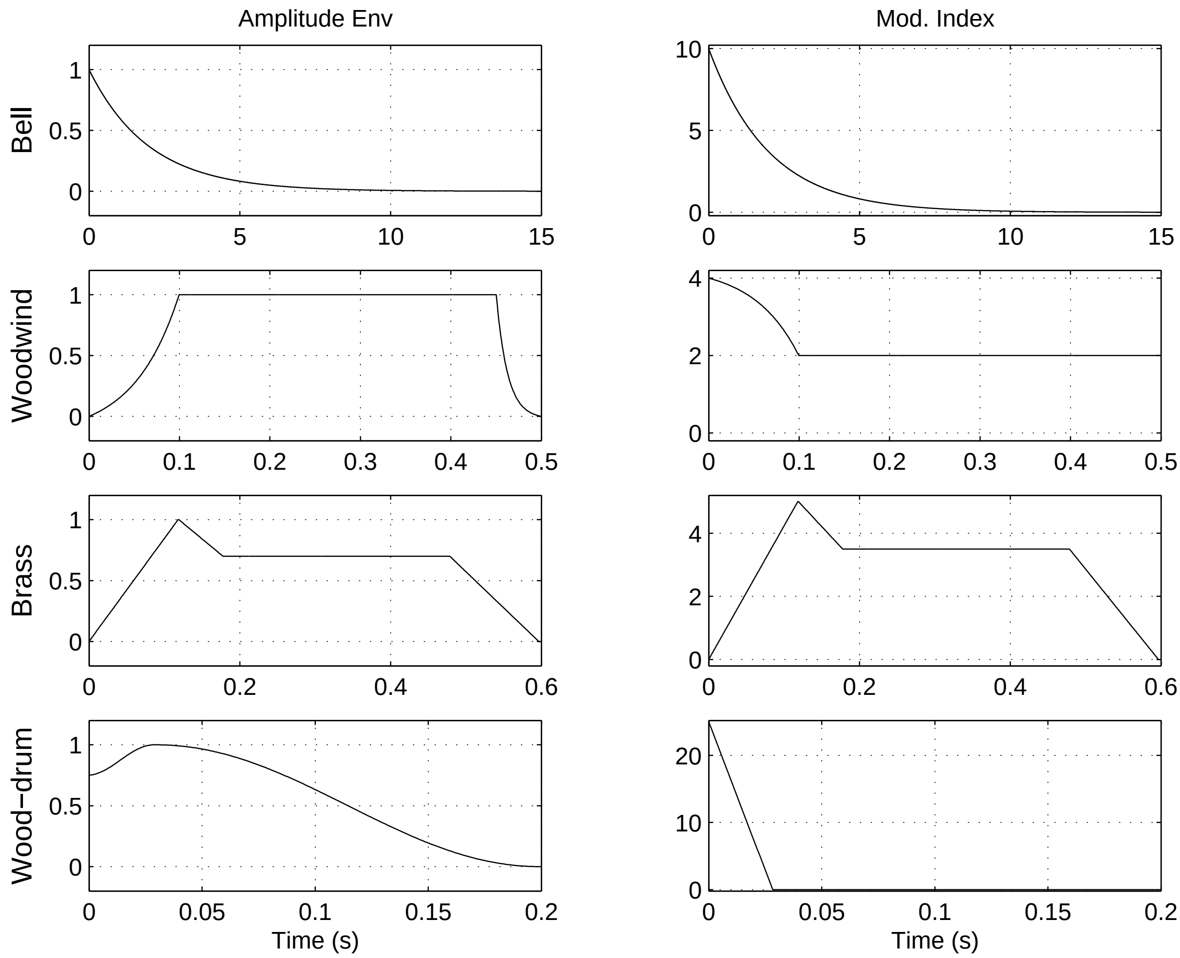


Figure 2: Envelopes for FM bell-like tones, wood-drum tones, brass-like tones and clarinet tones.

FM Instrument Parameters

	f_c	f_m	I_{MIN}	I_{MAX}
bell	110	220		10
	150	350		5
brass	f_0	f_0		2.66-5
clarinet	$3f_0$	$2f_0$	2	4
woodrum	80	55		25