Pressure, Power and Intensity

- The waveform of sound shows the time evolution of the pressure variations—its amplitude.

- Related to sound pressure are the
  1. sound power emitted by the source
  2. sound intensity measured some distance from the source.

- Power:
  - Watts (W) or Joules/second (J/s),
  - equivalent to the amount of energy consumed per unit of time.

- Sound power is analogous to the wattage rating of a light bulb—it’s a fixed quantity.

Intensity

- **Intensity** is the power per unit area,
  \[ \text{Intensity} = \frac{\text{Power}}{A}, \]
  expressed in Watts/square meter (W/m²).

- **Sound intensity** is
  - a measure of the power in a sound that actually contacts an area (e.g. eardrum);
  - a quantity influenced by environment surroundings/surfaces and interference from other sources;
  - analogous to the brightness of the light in a particular part of the room.

- Intensity is related to pressure squared:
  \[ I = \frac{p^2}{(\rho c)}, \]
  where \( \rho \) is the density of air (kg/m³), and \( c \) is the speed of sound (m/s).

Sound Range of Hearing

- Amplitude (pressure) range of hearing (humans)
  - Threshold of audibility: 0.00002 N/m²
  - Threshold of feeling (or pain!): 200 N/m²

- Sound intensity range (humans)
  - \( I_0 = 10^{-12} \) W/m² (threshold of audibility)
  - 1 W/m² (threshold of feeling)

- The intensity ratio between the sounds that bring pain to our ears and the weakest sounds we can hear is more than 10^{12}. 
Linear vs logarithmic scales.

- Human hearing is better measured on a logarithmic scale than a linear scale.
- On a linear scale,
  - a change between two values is perceived on the basis of the difference between the values;
  - e.g., a change from 1 to 2 would be perceived as the same amount of increase as from 4 to 5.
- On a logarithmic scale,
  - a change between two values is perceived on the basis of the ratio of the two values;
  - e.g., a change from 1 to 2 would be perceived as the same amount of increase as a change from 4 to 8 (also a ratio of 1:2).

![Figure 1: Moving one unit to the right increments by 1 on the linear scale and multiplies by a factor of 10 on the logarithmic scale.](image)

Decibels

- The decibel (dB) is a unit named after Alexander Graham Bell, known as a telecommunications pioneer.
- The decibel is a logarithmic scale, used to compare two quantities such as
  - the power gain of an amplifier;
  - the relative power of two sound sources.
- A decibel is defined as one tenth of a bel, $1 \text{B} = 10 \text{dB}$.

To convert from Bel to dB, multiply by 10.

- The decibel difference between two power levels $\Delta L$ is defined in terms of their power ratio $W_2/W_1$:
  $$\Delta L = 10 \log \frac{W_2}{W_1} \text{ dB}.$$  
- Since power is proportional to intensity, the ratio of two signals with intensities $I_1$ and $I_2$ is given in decibels by
  $$\Delta L = 10 \log \frac{I_2}{I_1} \text{ dB}.$$  

Sound Power and Intensity Level

- Though decibels are often used as absolute measurements, there is an implied fixed reference.
- The sound power level of a source is expressed using the threshold of audibility $W_0 = 10^{-12}$ as a reference:
  $$L_W = 10 \log \frac{W}{W_0} \text{ dB}.$$  
- The sound intensity level (SIL) at some distance from the source can be expressed in dB by comparing it to the reference $I_0 = 10^{-12} \text{ W/m}^2$:
  $$L_I = 10 \log \frac{I}{I_0} \text{ dB}.$$  

Sound pressure Level (SPL or $L_p$)

- Intensity is proportional to sound pressure amplitude squared
  $$I = p^2/(\rho c),$$  
  where $\rho c \approx 400$.
- For a plane wave travelling down a tube or spherical wave traveling outward from a source (when $\rho c = 400$):
  $$L_p = L_I.$$  
- This can be used to express SPL in dB by:
  $$L_p = 10 \log \frac{I}{I_0} = 10 \log \frac{p^2}{(\rho c I_0)} = 10 \log \frac{p^2}{(4 \times 10^{-10})} = 10 \log \left(\frac{p}{(2 \times 10^{-5})}\right)^2 = 20 \log \frac{p}{(2 \times 10^{-5})} = 20 \log p/p_0 \text{ dB}.$$  
  where $p_0 = 2 \times 10^{-5}$ is the threshold of hearing for amplitude of pressure variations.
Example SPL Levels

- The following is taken from Dan Levitin’s, *Your Brain on Music*.
  - 0 dB: a mosquito flying in a quiet room, ten feet away from your ears
  - 20 dB: a recording studio
  - 35 dB: a typical quiet office with the door closed and computer off
  - 50 dB: typical conversation
  - 75 dB: typical comfortable music listening level (headphones)
  - 100-105 dB: Classical music concert during loud passages; the highest level of some portable music players
  - 110 dB: A jackhammer 3 feet away
  - 120 dB: A jet engine heard on the runway from 300 ft away; typical rock concert
  - 126-130 dB: Threshold of pain and damage; a rock concert by the Who
  - 180 dB: Space shuttle launch

Increasing distance from a source

- A point source is one from which a spherical wave radiates in free space (equally in all directions) and in which:
  - the intensity decreases by $1/r^2$ with an increase in radius $r$ (the surface area of a sphere is $4\pi r^2$)
  - the pressure decreases by $1/r$ (since pressure is proportional to the square root of intensity).
- In actual practice, sound sources wouldn’t radiate so symmetrically as there is likely to be interference from other reflective objects.

SIL at a Distance from the Source

- **Problem**: Given the sound power level at a source, what is the SIL at a distance 1 m from that source in free field?
- **Solution**: If a given amount of sound power is distributed over the surface of an expanding sphere, the intensity is $I = \frac{W}{4\pi r^2}$.
  - The intensity at 1 m from the source is given by $I = \frac{W}{4\pi r^2} = \frac{W}{4\pi}$.
  - The corresponding SIL is $L_I = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{W}{4\pi}\right)$.
  - or 11 dB less than the sound power level at the source.

Sound Level with a Doubling of Distance

- **Problem**: How does the SIL change with a doubling of distance?
- **Solution**: We know that the intensity will drop by $1/r^2$ which is a factor of $1/2^2$ for a doubling of distance. Thus,
  \[
  L_I = 10\log\left(\frac{1}{2^2} \times \frac{I}{I_0}\right) = 10\log\left(I/I_0\right) + 10\log(1/2^2) = \text{SIL} - 10\log(2^2) = \text{SIL} - 20\log(2) = \text{SIL} - 6,
  \]
  corresponding to a decrease of 6 dB.
Multiple Sources

- When there are multiple independent sound sources, the total power emitted is the sum of the power from each source.
- **Problem**: By how much would the sound level increase when two independent sources with equal power sound simultaneously?
- **Solution**: The sound power level (at the source) would double, yielding the sound power level
  \[ L_{\text{W}} = 10 \log(2W/W_0) \]
  \[ = 10 \log(W/W_0) + 10 \log(2) \]
  \[ = 10 \log(W/W_0) + 3 \text{ dB}, \]
  which corresponds to an increase of 3 dB.
- This is the case most of the time. However, there is an exception...

Multiple Correlated Sound Sources

- If the sound sources emit waveforms that are strongly correlated, there will be interference.
- When two waves of the same frequency reach the same point, they may interfere destructively or constructively.
- The resulting overall amplitude can be anywhere from
  - 0 (complete destructive interference) to
  - the sum of the individual amplitudes (complete constructive interference).

Complete Constructive Interference

- Summing two in-phase sinusoids at the same frequency and amplitude doubles the pressure, yielding
  \[ SPL = 20 \log \frac{2p}{p_0} \]
  \[ = 20 \log \frac{p}{p_0} + 20 \log(2) \]
  \[ = 20 \log \frac{p}{p_0} + 6, \]
  which is an increase of 6 dB.

Listening to Sound Level’s

- **db.pd**
- What does an increase in 1 dB sound like? (not much impact)
- A 6 dB difference (a factor of 4 in power or 2 in pressure) is easy to hear, but would not have a tremendous musical impact.
Loudness Level and Frequency

- Loudness is a subjective quality that depends on SPL, but also on frequency and other factors.
- Dependency on frequency is shown by Fletcher and Munson’s equal loudness curves: the required SPL for different frequencies to sound equally loud.

![Equal-loudness curves for pure tones (expressed in phons).](image)

- The equal-loudness curves are expressed in **phons**:
  - phon ≜ SPL in dB at a frequency of 1000 Hz.