On the Transfer Function of the Piecewise (Cylindrical) Model of the Vocal Tract

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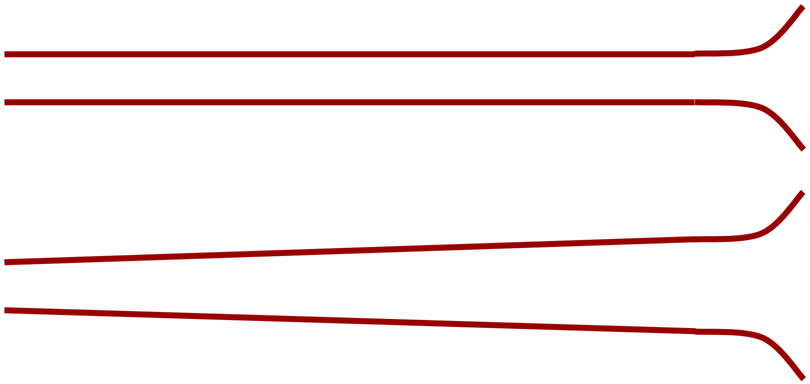
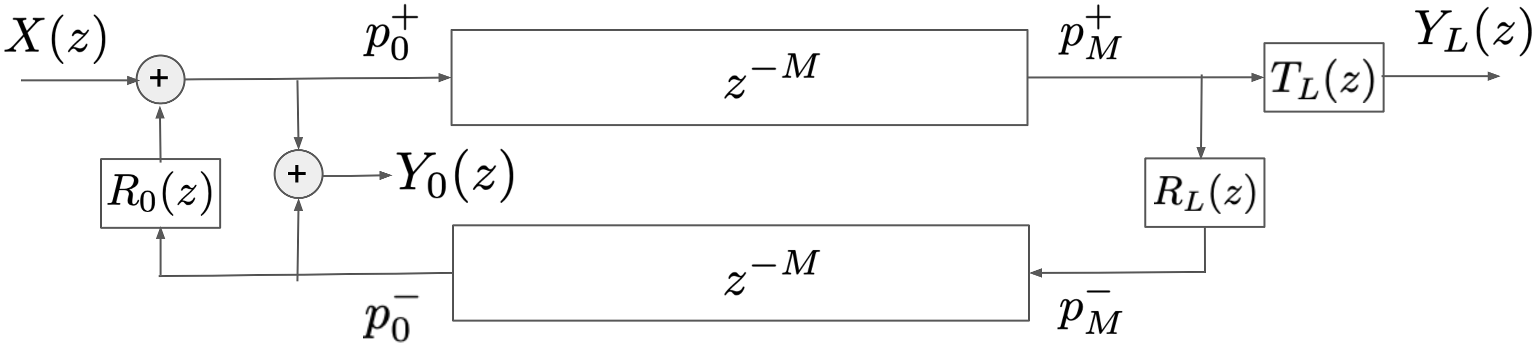
Outline

- One-Dimensional Waveguide Acoustic Tube Model
- Transfer Function (1): cylinders and cones, non-polynomial form
- Cylindrical Sections
- Transfer Function (2): ratio of polynomials
  - Boundaries with scalar loss
  - Relationship to LPC
- Transfer Function (3):
  - Boundaries with frequency-dependent loss

Cylindrical and Conical Acoustic Tubes



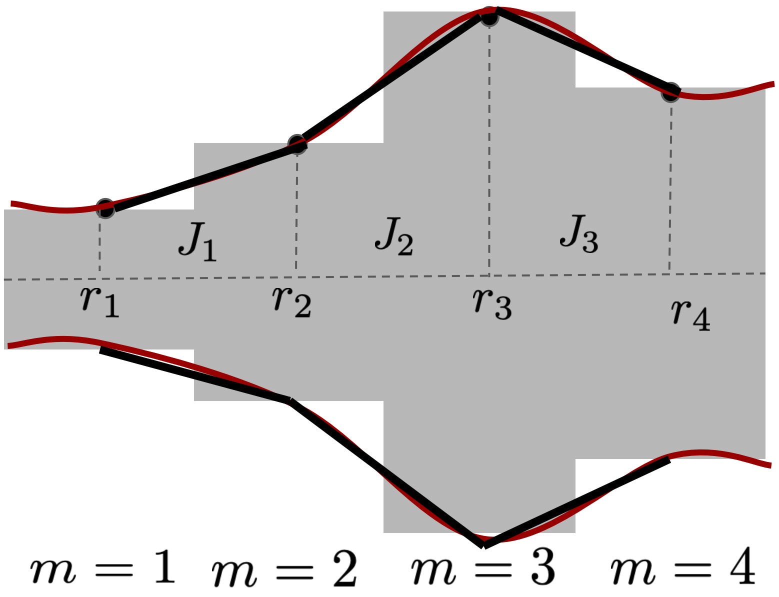
Waveguide Model



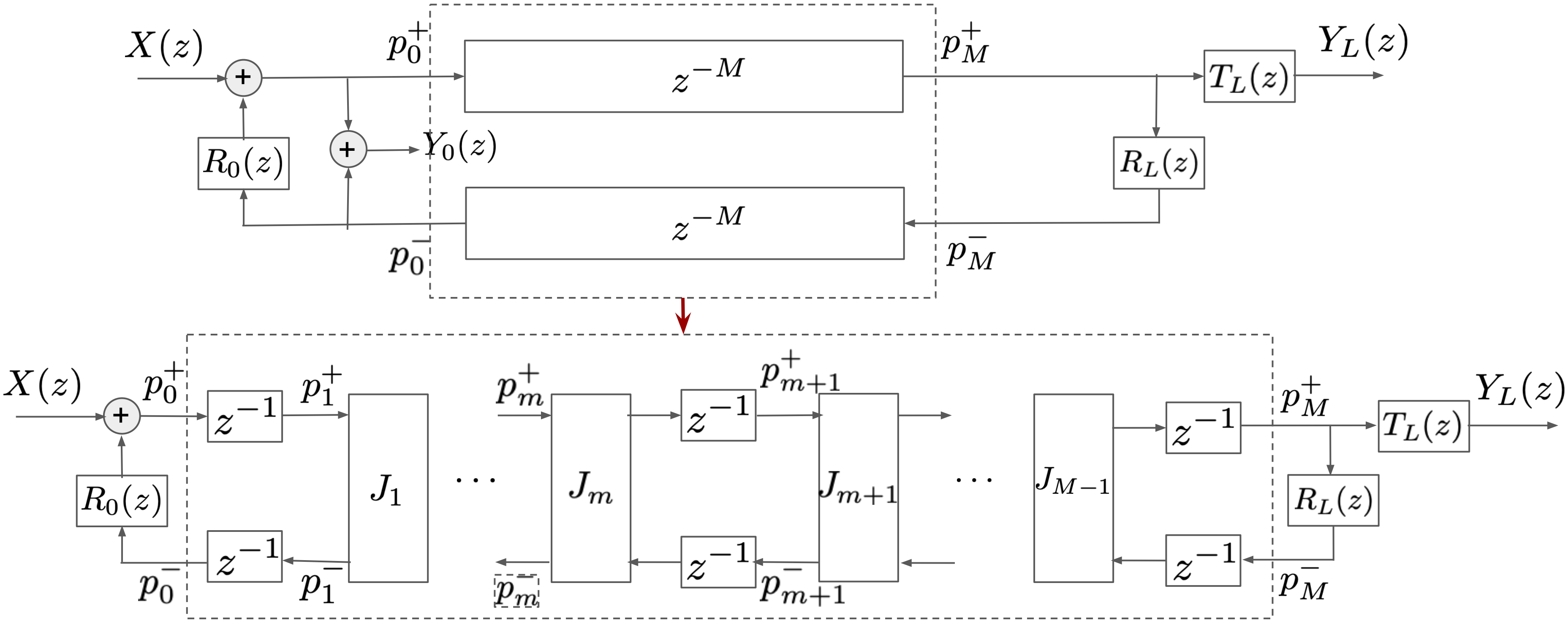
Acoustic Tube with Varying Cross-Sectional Area

Radii sampled/measured at regular spatial intervals

allows for a piecewise approximation

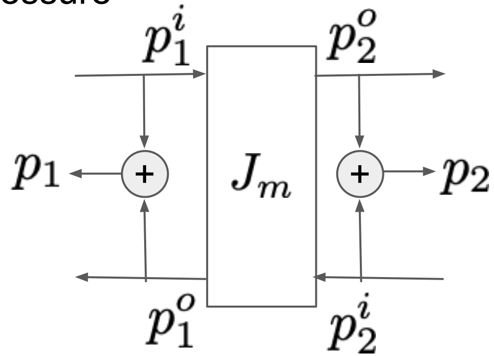


Piecewise Waveguide Model



Two-Port Scattering Junction (1-D wave propagation)

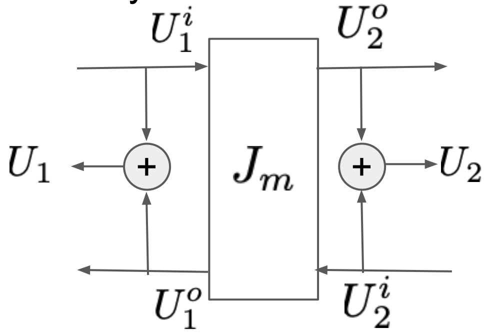
Pressure



$$U_n^i = \frac{p_n^i}{Z_n(\omega)}$$

$$U_n^o = -\frac{p_n^o}{Z_n^*(\omega)}$$

Volume velocity



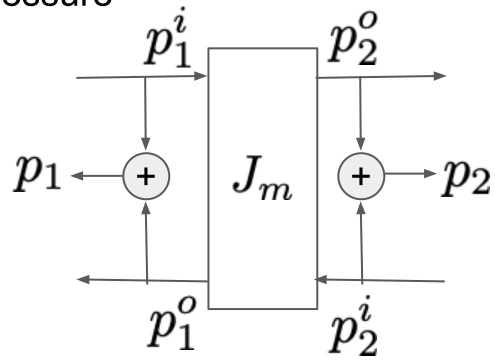
Characteristic wave impedance:

- **cone:** a complex function of frequency:
  - **diverging**: $Z(l,\omega) = \frac{\rho c}{S} \cdot \frac{j\omega}{j\omega + c/l}$ **converging**: $Z^*(l,\omega) = \frac{\rho c}{S} \cdot \frac{j\omega}{j\omega - c/l}$
- **cylinder:** a scalar value:

$$Z(l,\omega) = \frac{\rho c}{S} \cdot \frac{\cancelto{1}{j\omega}}{j\omega + \cancelto{0}{c/l}} \longrightarrow Z = \frac{\rho c}{S}$$

Two-Port Scattering Junction

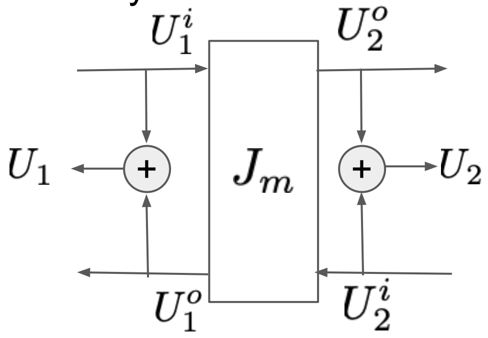
Pressure



$$U_n^i = \frac{p_n^i}{Z_n(\omega)}$$

$$U_n^o = -\frac{p_n^o}{Z_n^*(\omega)}$$

Volume velocity



The law for conservation of mass and momentum:

- **pressure** at junction equals pressure on each port

$$p_J = p_1 = p_2$$

$$\boxed{p_1^i + p_1^o = p_2^i + p_2^o}$$

- **volume velocity** on the each port sums to zeros

$$U_1 + U_2 = 0$$

$$U_1^i + U_1^o = -\left(U_2^i + U_2^o\right)$$

$$\boxed{\frac{p_1^i}{Z_1} - \frac{p_1^o}{Z_1^*} = -\left(\frac{p_2^i}{Z_2} - \frac{p_2^o}{Z_2^*}\right)}$$

Relating Port Inputs and Outputs

System of equations:

$$p_1^i + p_1^o = p_2^i + p_2^o$$

$$\frac{p_1^i}{Z_1} - \frac{p_1^o}{Z_1^*} = -\left(\frac{p_2^i}{Z_2} - \frac{p_2^o}{Z_2^*}\right)$$

Matrix representation:

$$\mathbf{C}\begin{bmatrix} p_1^i \\ p_1^o \end{bmatrix} = \mathbf{D}\begin{bmatrix} p_2^i \\ p_2^o \end{bmatrix}$$

Pressure on either side of the junction:

$$\begin{bmatrix} p_1^i \\ p_1^o \end{bmatrix} = \mathbf{C}^{-1}\mathbf{D}\begin{bmatrix} p_2^i \\ p_2^o \end{bmatrix}$$

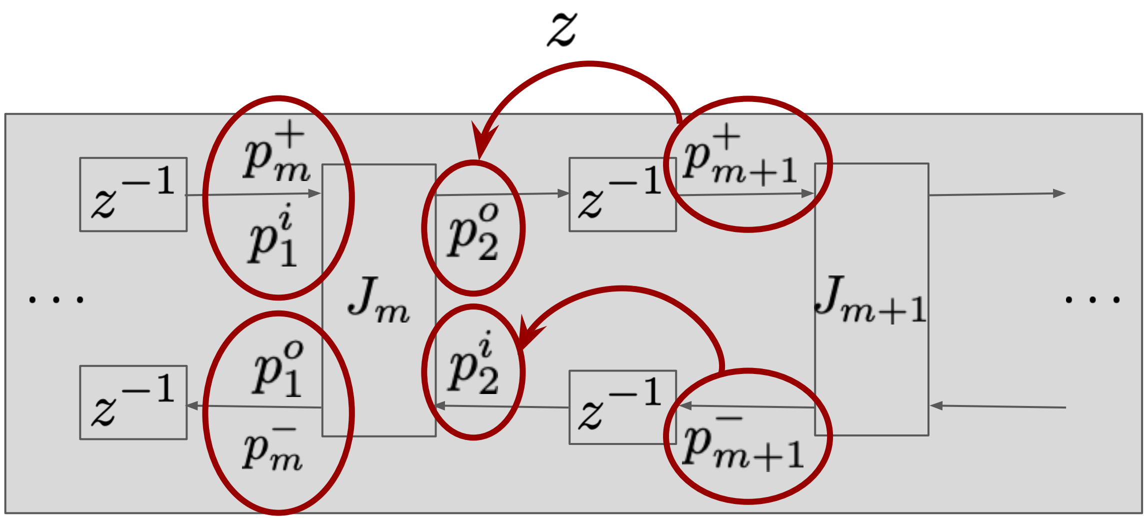
$$\mathbf{C} = \begin{bmatrix} 1 & 1 \\ \frac{1}{Z_1} & -\frac{1}{Z_1^*} \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 1 \\ -\frac{1}{Z_2} & \frac{1}{Z_2^*} \end{bmatrix}$$

$$\mathbf{C}^{-1} = \frac{1}{\frac{1}{Z_1^*} + \frac{1}{Z_1}}\begin{bmatrix} \frac{1}{Z_1^*} & 1 \\ \frac{1}{Z_1} & -1 \end{bmatrix}$$

Port I/O and Right/Left Traveling Waves

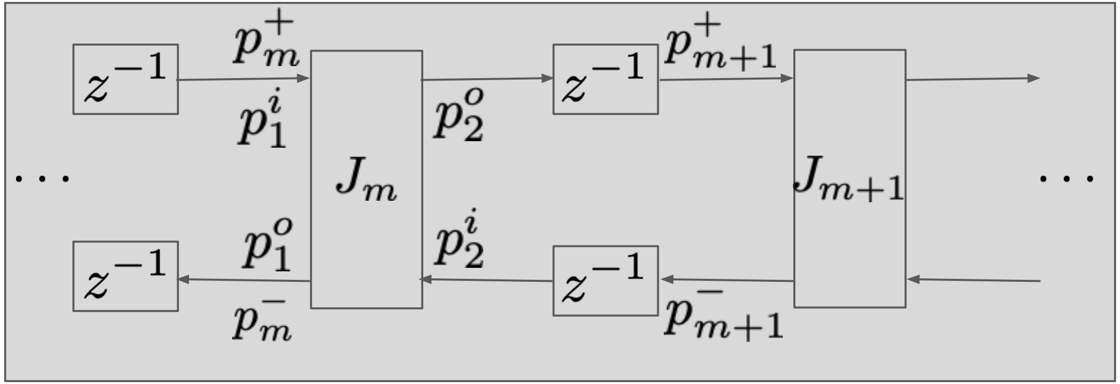
$$\underbrace{\begin{bmatrix}p_1^i\\ p_1^o\end{bmatrix}}_{\underbrace{\begin{bmatrix}p_m^+\\ p_m^-\end{bmatrix}}_{\mathbf{p}_m}} = \mathbf{C}^{-1}\mathbf{D}\underbrace{\begin{bmatrix}p_2^i\\ p_2^o\end{bmatrix}}_{\underbrace{\begin{bmatrix}p_{m+1}^- z^{-1}\\ p_{m+1}^+ z\end{bmatrix}}_{\begin{bmatrix}0 & z^{-1}\\ z & 0\end{bmatrix}\underbrace{\begin{bmatrix}p_{m+1}^+\\ p_{m+1}^-\end{bmatrix}}_{\mathbf{p}_{m+1}}}}$$



Scattering Matrix

$$\underbrace{\begin{bmatrix} p_1^i \\ p_1^o \end{bmatrix}} = \mathbf{C}^{-1}\mathbf{D} \underbrace{\begin{bmatrix} p_2^i \\ p_2^o \end{bmatrix}}$$

$$\underbrace{\begin{bmatrix} p_m^+ \\ p_m^- \end{bmatrix}}_{\mathbf{p}_m} = \underbrace{\left(\mathbf{C}^{-1}\mathbf{D}\right)_m \begin{bmatrix} 0 & z^{-1} \\ z & 0 \end{bmatrix}}_{\mathbf{A}_m} \underbrace{\begin{bmatrix} p_{m+1}^+ \\ p_{m+1}^- \end{bmatrix}}_{\mathbf{p}_{m+1}}$$



$$\mathbf{C}^{-1}\mathbf{D} = \frac{1}{\frac{1}{Z_1^*} + \frac{1}{Z_1}} \begin{bmatrix} \frac{1}{Z_1^*} & 1 \\ \frac{1}{Z_1} & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -\frac{1}{Z_2} & \frac{1}{Z_2^*} \end{bmatrix} = \begin{bmatrix} \frac{Z_1\left(Z_2 - Z_1^*\right)}{Z_2\left(Z_1 + Z_1^*\right)} & \frac{Z_1\left(Z_2^* - Z_1^*\right)}{Z_2^*\left(Z_1 + Z_1^*\right)} \\ \frac{Z_1^*\left(Z_2 - Z_1\right)}{Z_2\left(Z_1 + Z_1^*\right)} & \frac{Z_1^*\left(Z_2^* - Z_1\right)}{Z_2^*\left(Z_1 + Z_1^*\right)} \end{bmatrix}$$

"Chain" Scattering Matrix

Relationship between traveling waves in adjacent sections:

$$\mathbf{p}_m = \mathbf{A}_m \underbrace{\mathbf{p}_{m+1}}_{\mathbf{A}_{m+1}\underbrace{\mathbf{p}_{m+2}}_{\mathbf{A}_{m+2}\mathbf{p}_{m+3}}}$$

Scattering matrix (single junction):

$$\mathbf{A}_m = \left(\mathbf{C}^{-1}\mathbf{D}\right)_m \begin{bmatrix} 0 & z^{-1} \\ z & 0 \end{bmatrix}$$

… and between first and final sections: $\mathbf{p}_1 = \underbrace{\left(\prod_{m=1}^{M-1} \mathbf{A}_m\right)}_{\mathbf{P}_{M-1}} \mathbf{p}_M$

"Chain" Scattering Matrix: $\mathbf{P}_{M-1} = \begin{bmatrix} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{bmatrix}$

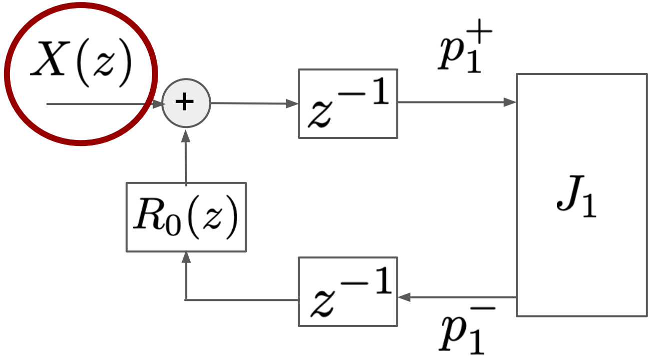
Toward the Transfer Function

The transfer function is the (spectral) ratio of the model **output** to the **input**:

$$H_L(z) = \frac{Y_L(z)}{X(z)} = \frac{p_M^+ T_L(z)}{X(z)}$$

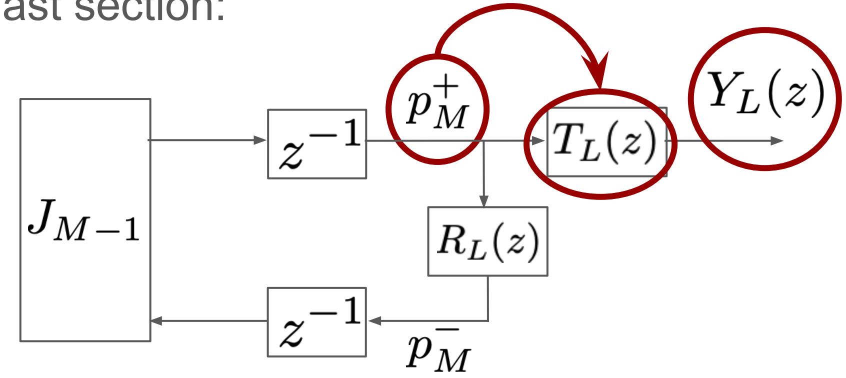
express as a function of p_M^+

first section:



$\cdots$

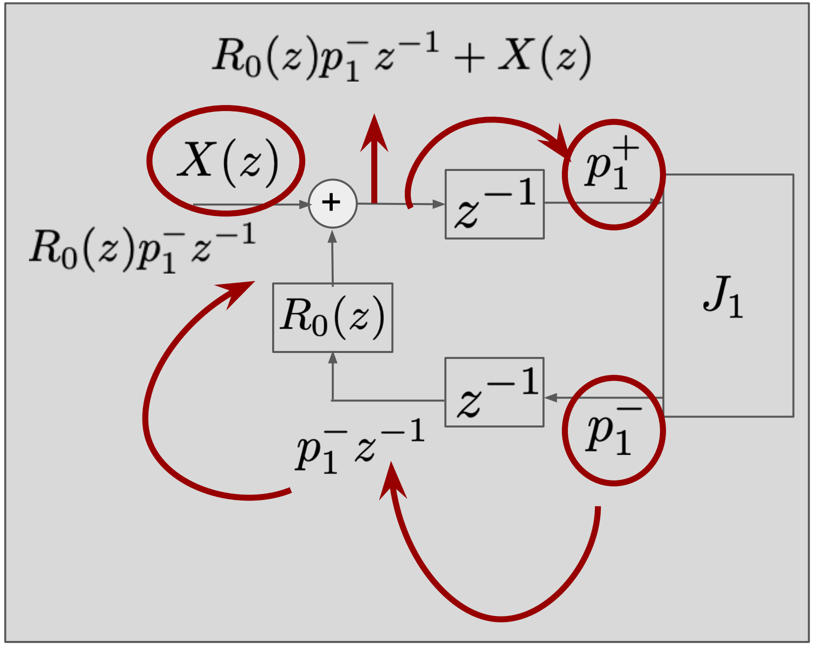
last section:



The Model Input

Input as a function of traveling waves in the **first section:**

First section:



$$z\, p_1^+ = \left(R_0(z)p_1^- z^{-1} + X(z)\right) \cancel{z^{-1} z}$$

$$p_1^+ z = R_0(z)p_1^- z^{-1} + X(z)$$

$$X(z) = p_1^+ z - R_0(z) p_1^- z^{-1}$$

Next, express as a function of traveling waves in the **final section**

Model Input

Input as a function of traveling waves in the **first** section:

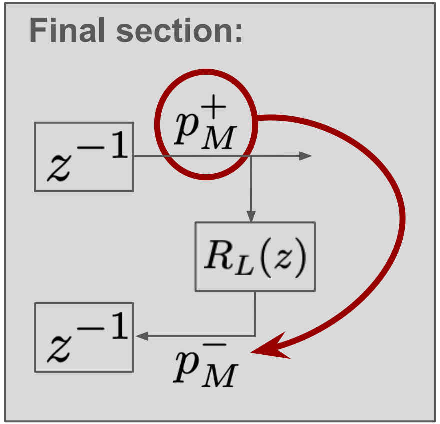
$$\mathbf{p}_1 = \mathbf{P}_{M-1}\mathbf{p}_M$$

$$\begin{bmatrix} p_1^+ \\ p_1^- \end{bmatrix} = \underbrace{\begin{bmatrix} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{bmatrix}}_{\text{“chain” scattering matrix}} \begin{bmatrix} p_M^+ \\ p_M^- \end{bmatrix}$$

$$X(z) = \underbrace{p_1^+}_{P_{1,1}p_M^+ + P_{1,2}p_M^-} z - R_0(z)\underbrace{p_1^-}_{P_{2,1}p_M^+ + P_{2,2}p_M^-} z^{-1}$$

Input as a function of traveling waves in the **final** section:

$$X(z) = \left(P_{1,1}p_M^+ + P_{1,2}\underbrace{p_M^-}_{R_L(z)p_M^+}\right) z - R_0(z)\left(P_{2,1}p_M^+ + P_{2,2}\underbrace{p_M^-}_{R_L(z)p_M^+}\right) z^{-1}$$



Input as a function of **right** traveling wave in the **final** section:

$$X(z) = p_M^+\left(P_{1,1} + P_{1,2}R_L(z)\right) z - R_0(z)p_M^+\left(P_{2,1} + P_{2,2}R_L(z)\right) z^{-1}$$

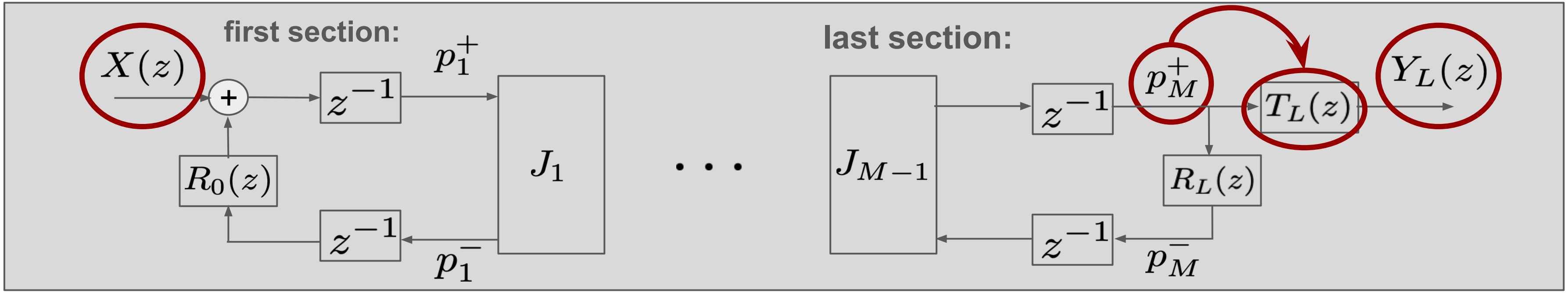
Transfer Function (1): cone/cylinder, non-polynomial form

The transfer function is the (spectral) ratio of the model **output** to the **input**:

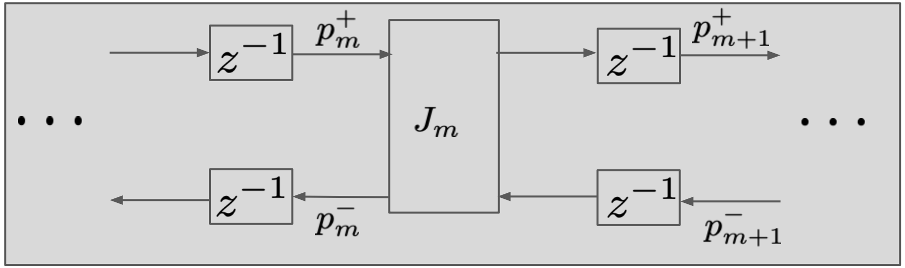
$$H_L(z) = \frac{Y_L(z)}{X(z)} = \underbrace{\frac{p_M^+ T_L(z)}{X(z)}}_{\text{express as a function of } p_M^+}$$

$$= \frac{\cancel{p_M^+} T_L(z)}{\cancel{p_M^+}\left(P_{1,1} + P_{1,2}R_L(z)\right) z - R_0(z)\cancel{p_M^+}\left(P_{2,1} + P_{2,2}R_L(z)\right) z^{-1}} \times \frac{z^{-1}}{z^{-1}}$$

$$= \frac{T_L(z) z^{-1}}{P_{1,1} + P_{1,2}R_L(z) - R_0(z)\left(P_{2,1} + P_{2,2}R_L(z)\right) z^{-2}}$$



Junctions for Cylindrical Sections



$$\boxed{Z_n = \frac{\rho c}{S_n}}$$

$$\mathbf{C}^{-1}\mathbf{D} = \begin{bmatrix} \frac{Z_1\left(Z_2 - Z_1^*\right)}{Z_2\left(Z_1 + Z_1^*\right)} & \frac{Z_1\left(Z_2^* + Z_1^*\right)}{Z_2^*\left(Z_1 + Z_1^*\right)} \\ \frac{Z_1^*\left(Z_2 + Z_1\right)}{Z_2\left(Z_1 + Z_1^*\right)} & \frac{Z_1^*\left(Z_2^* - Z_1\right)}{Z_2^*\left(Z_1 + Z_1^*\right)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{Z_2 - Z_1}{2Z_2} & \frac{Z_2 + Z_1}{2Z_2} \\ \frac{Z_2 - Z_1}{2Z_2} & \frac{Z_2 + Z_1}{2Z_2} \end{bmatrix}$$

$$= \frac{1}{2S_1}\begin{bmatrix} S_1 - S_2 & S_1 + S_2 \\ S_1 + S_2 & S_1 - S_2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} p_m^+ \\ p_m^- \end{bmatrix}}_{\mathbf{p}_m} = \underbrace{\left(\mathbf{C}^{-1}\mathbf{D}\right)_m \begin{bmatrix} 0 & z^{-1} \\ z & 0 \end{bmatrix}}_{\mathbf{A}_m} \underbrace{\begin{bmatrix} p_{m+1}^+ \\ p_{m+1}^- \end{bmatrix}}_{\mathbf{p}_{m+1}}$$

for m^{th} scattering junction: $\left(\mathbf{C}^{-1}\mathbf{D}\right)_m = \frac{1}{2S_m}\begin{bmatrix} S_m - S_{m+1} & S_m + S_{m+1} \\ S_m + S_{m+1} & S_m - S_{m+1} \end{bmatrix}$

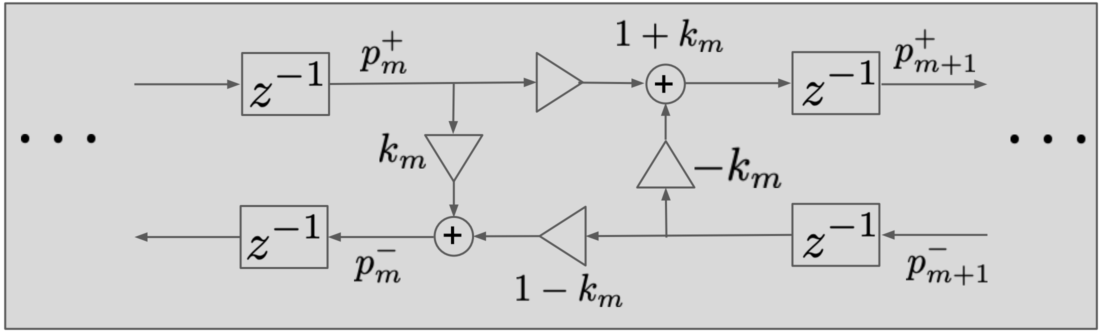
Reflection Coefficients

$$\left(\mathbf{C}^{-1}\mathbf{D}\right)_m = \frac{1}{2S_m}\begin{bmatrix} S_m - S_{m+1} & S_m + S_{m+1} \\ S_m + S_{m+1} & S_m - S_{m+1} \end{bmatrix}$$
$$= \frac{1}{1+k_m}\begin{bmatrix} k_m & 1 \\ 1 & k_m \end{bmatrix}$$

Scattering Matrix (cylinders):

$$\mathbf{A}_m = \left(\mathbf{C}^{-1}\mathbf{D}\right)_m \begin{bmatrix} 0 & z^{-1} \\ z & 0 \end{bmatrix} = \frac{1}{1+k_m}\begin{bmatrix} k_m & 1 \\ 1 & k_m \end{bmatrix}\begin{bmatrix} 0 & z^{-1} \\ z & 0 \end{bmatrix} = \frac{z}{1+k_m}\begin{bmatrix} 1 & k_m z^{-2} \\ k_m & z^{-2} \end{bmatrix}$$

Kelly-Lochbaum Scattering Junction



reflection coefficient

$$k_m = \frac{S_m - S_{m+1}}{S_m + S_{m+1}}$$

“Chain” Scattering Matrix $(N = M - 1$ junctions)

$$\mathbf{A}_m = \frac{z}{1+k_m}\begin{bmatrix} 1 & k_m z^{-2} \\ k_m & z^{-2} \end{bmatrix}$$

$$\mathbf{P}_N = \prod_{m=1}^{N} \mathbf{A}_m = \frac{z}{1+k_1}\begin{bmatrix} 1 & k_1 z^{-2} \\ k_1 & z^{-2} \end{bmatrix} \frac{z}{1+k_2}\begin{bmatrix} 1 & k_2 z^{-2} \\ k_2 & z^{-2} \end{bmatrix} \cdots \frac{z}{1+k_N}\begin{bmatrix} 1 & k_N z^{-2} \\ k_N & z^{-2} \end{bmatrix}$$

$$= \frac{z^N}{\prod_{m=1}^{N}(1+k_m)} \underbrace{\begin{bmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \end{bmatrix}}_{\mathbf{K}_N}$$

Matrix elements are polynomials in z

$$K_{1,1} = c_0 + c_2 z^{-2} + \cdots + c_{2(N-1)} z^{-2(N-1)}$$

$$K_{2,1} = d_0 + d_2 z^{-2} + \cdots + d_{2(N-1)} z^{-2(N-1)}$$

$$K_{1,2} = d_{2(N-1)} z^{-2} + \cdots + d_2 z^{-2(N-1)} + d_0 z^{-2N}$$

$$K_{2,2} = c_{2(N-1)} z^{-2} + \cdots + c_2 z^{-2(N-1)} + c_0 z^{-2N}$$

Right column:

coefficients in reverse order

Polynomial Coefficients

Coefficient vectors have the form:

$$\mathbf{c}_N = \begin{bmatrix} c_0 \\ 0 \\ c_2 \\ 0 \\ \vdots \\ c_{2(N-1)} \\ 0 \end{bmatrix} \mathbf{d}_N = \begin{bmatrix} d_0 \\ 0 \\ d_2 \\ 0 \\ \vdots \\ d_{2(N-1)} \\ 0 \end{bmatrix}$$

Initial coefficients:

$$c_0 = 1 \qquad d_0 = k_1$$

Matrix elements are polynomials in z

$$K_{1,1} = c_0 + c_2 z^{-2} + \cdots + c_{2(N-1)} z^{-2(N-1)}$$

$$K_{2,1} = d_0 + d_2 z^{-2} + \cdots + d_{2(N-1)} z^{-2(N-1)}$$

$$K_{1,2} = d_{2(N-1)} z^{-2} + \cdots + d_2 z^{-2(N-1)} + d_0 z^{-2N}$$

$$K_{2,2} = c_{2(N-1)} z^{-2} + \cdots + c_2 z^{-2(N-1)} + c_0 z^{-2N}$$

Coefficients are recursively defined:

$$\mathbf{c}_N = \begin{bmatrix} \mathbf{c}_{N-1} \\ 0 \\ 0 \end{bmatrix} + k_N \begin{bmatrix} 0 \\ \tilde{\mathbf{d}}_{N-1} \\ 0 \end{bmatrix}$$

$$\mathbf{d}_N = \begin{bmatrix} \mathbf{d}_{N-1} \\ 0 \\ 0 \end{bmatrix} + k_N \begin{bmatrix} 0 \\ \tilde{\mathbf{c}}_{N-1} \\ 0 \end{bmatrix}$$

~ denotes order of elements is reversed

Transfer Function (2): scalar boundaries

Recall first representation:

$$H_L(z) = \frac{Y_L(z)}{X(z)} = \frac{\cancel{T_L(z)}z^{-1}}{P_{1,1} + P_{1,2}R_L(z) - R_0(z)\left(P_{2,1} + P_{2,2}R_L(z)\right) z^{-2}}$$

$$\mathbf{P}_N = \begin{bmatrix} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{bmatrix} = \frac{\cancel{z^N}}{z^{-N}\prod\limits_{m=1}^{N}(1+k_m)} \underbrace{\begin{bmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \end{bmatrix}}_{\mathbf{K}_N}$$

$$H_L(z) = \frac{\overbrace{z^{-(N+1)}}^{\text{pure delay}} \overbrace{\prod_{m=1}^{N}(1+k_m)}^{\text{scalar}}}{\underbrace{K_{1,1} + K_{1,2}R_L - R_0\left(K_{2,1} + K_{2,2}R_L\right) z^{-2}}} = \frac{B(z)}{A(z)}$$

$$A(z) = a_0 z^{-0} + a_1 z^{-1} + \cdots + a_{2(N+1)} z^{-2(N+1)}$$

Transfer Function (2): scalar boundaries

Denominator polynomial:

$$A(z) = a_0 z^{-0} + a_1 z^{-1} + \cdots + a_{2(N+1)} z^{-2(N+1)}$$

Coefficient (column) vector:

$$\mathbf{A}_N = \mathbf{C}_N \mathbf{R}$$

$$\mathbf{C}_N = \begin{bmatrix} \mathbf{c}_N & 0 & 0 & 0 \\ 0 & \tilde{\mathbf{d}}_N & 0 & 0 \\ 0 & 0 & \mathbf{d}_N & 0 \\ 0 & 0 & 0 & \tilde{\mathbf{c}}_N \end{bmatrix} = \begin{bmatrix} c_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & \mathbf{0} & \mathbf{0} \\ c_2 & d_{2(N-1)} & d_0 & \mathbf{0} \\ 0 & 0 & 0 & 0 \\ c_4 & d_{2(N-2)} & d_2 & c_{2(N-1)} \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ c_{2(N-1)} & d_2 & d_{2(N-2)} & c_4 \\ 0 & 0 & 0 & 0 \\ \mathbf{0} & d_0 & d_{2(N-1)} & c_2 \\ \mathbf{0} & \mathbf{0} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & c_0 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 1 \\ R_L \\ -R_0 \\ -R_0 R_L \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -R_0 \\ R_0 \end{bmatrix} \qquad \mathbf{A}_N = \begin{bmatrix} 1 \\ 0 \\ a_2 \\ \vdots \\ 0 \\ a_{2N} \\ 0 \\ R_0 \end{bmatrix}$$

Relationship to LPC

Transfer Function:

$$H_L(z) = \frac{Y_L(z)}{X(z)} = \frac{z^{-(N+1)} \prod_{m=1}^{N} (1 + k_m)}{1 + \sum_{i=1}^{2(N+1)} a_i z^{-i}} = \frac{B(z)}{A(z)}$$

defined by coefficients vector $\mathbf{A}_N$

0

Difference Equation:

$$y(n) = \prod_{m=1}^{N} (1 + k_m) \cancel{x(n - (N+1))} - \sum_{i=1}^{2(N+1)} a_i y(n-i)$$

Input impulse (unit step function): $x(n) = u(n) = 1, 0, 0, 0, \ldots$

Impulse Response:

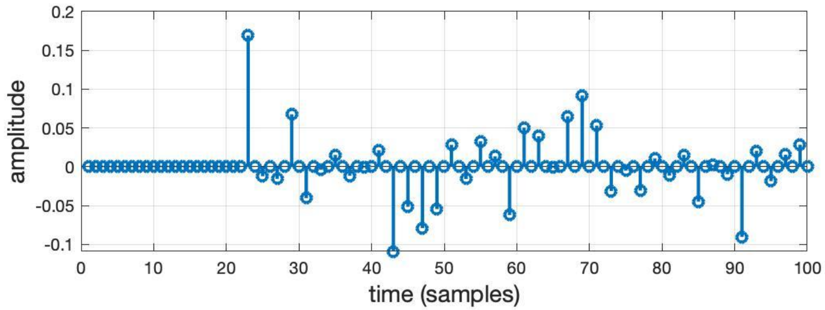
$$h(n) = \begin{cases} \mathbf{0}, & \textbf{for } \boldsymbol{n < N+1} \\ \boldsymbol{\prod_{m=1}^{N} (1 + k_m)}, & \textbf{for } \boldsymbol{n = N+1} \\ -\sum_{i=1}^{2(N+1)} a_i h(n-i), & \text{for } n > N+1. \end{cases}$$

Relationship to LPC

Coefficient vector $\mathbf{A}_N$ is strongly related to the LPC coefficients estimated from the impulse response of $H_L(z)$.

Impulse Response:

$$h(n) = \begin{cases} 0, & \text{for } n < N+1 \\ \prod_{m=1}^{N} (1 + k_m), & \text{for } n = N+1 \\ -\sum_{i=1}^{2(N+1)} a_i h(n-i), & \text{for } n > N+1. \end{cases}$$

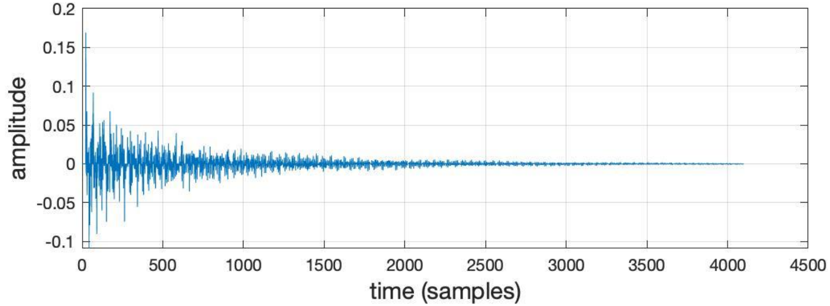


Linear Prediction:

finds coefficients $\hat{a}_i$ such that

$$\underbrace{\hat{x}(n)}_{\text{predicted signal}} = -\sum_{i=1}^{\overbrace{p}^{\text{order}}} \hat{a}_i x(n-i)$$

error: $e(n) = x(n) - \hat{x}(n)$ 0



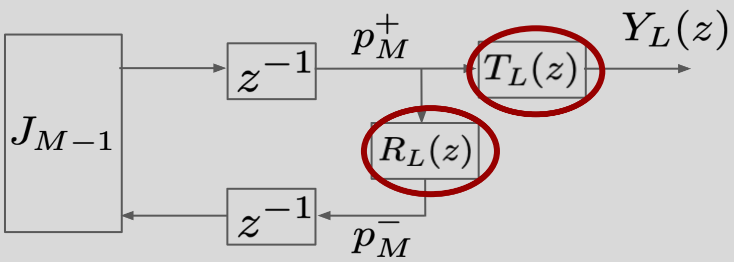
If order $p = 2(N+1)$ and $x(n) = h(n)$

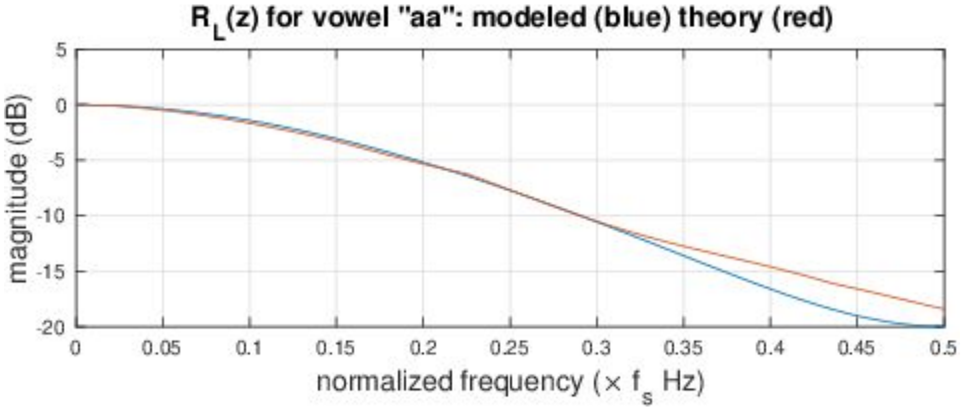
then $\hat{h}(n) = h(n)$ and $\hat{a}_i = a_i$

$$\mathbf{A}_N = \mathbf{A}_p$$ LPC estimated coefficients vector

Frequency-Dependent Lip Reflection

last section:





Reflection function at the lips:

$$R_L(z) = \frac{B_L(z)}{A_L(z)} = -\frac{(b_L)_0 + (b_L)_1 z^{-1} + (b_L)_2 z^{-2}}{1 + (a_L)_1 z^{-1} + (a_L)_2 z^{-2}}$$

$$\left.\begin{aligned} \mathbf{B}_L &= \begin{bmatrix}(b_L)_0 & (b_L)_1 & (b_L)_2\end{bmatrix} \\ \mathbf{A}_L &= \begin{bmatrix}(a_L)_0 & (a_L)_1 & (a_L)_2\end{bmatrix}\end{aligned}\right\} \text{lip reflection coefficient vectors}$$

Transmission function (amplitude complementary): $T_L(z) = 1 + R_L(z) = \dfrac{A_L(z) + B_L(z)}{A_L(z)}.$

Transfer Function (3): frequency-dependent boundaries

Transfer function (2): scalar boundaries:

$$H_L(z) = \frac{z^{-(N+1)} \prod_{m=1}^{N} (1 + k_m)}{K_{1,1} + K_{1,2} R_L - R_0 \left(K_{2,1} + K_{2,2} R_L\right) z^{-2}}$$

$$R_L(z) = \frac{B_L(z)}{A_L(z)}$$

Transfer function (3): frequency-dependent boundaries:

$$\hat{H}_L(z) = \frac{T_L(z) z^{-(N+1)} \prod_{m=1}^{N} (1 + k_m)}{K_{1,1} + K_{1,2} \frac{B_L(z)}{A_L(z)} - R_0 \left(K_{2,1} + K_{2,2} \frac{B_L(z)}{A_L(z)}\right) z^{-2}} \times \frac{A_L(z)}{A_L(z)}$$

$$= \frac{(A_L(z) + B_L(z)) z^{-(N+1)} \prod_{m=1}^{N} (1 + k_m)}{K_{1,1} A_L(z) + K_{1,2} B_L(z) - R_0 \left(K_{2,1} A_L(z) + K_{2,2} B_L(z)\right) z^{-2}} = \frac{\hat{B}(z)}{\hat{A}(z)}$$

$$T_L(z) = 1 + R_L(z)$$
$$= \frac{A_L(z) + B_L(z)}{A_L(z)}$$

Transfer Function (3): numerator polynomial

Transfer function (3):

$$\hat{H}_L(z) = \frac{(A_L(z) + B_L(z))z^{-(N+1)} \prod_{m=1}^{N}(1+k_m)}{K_{1,1}A_L(z) + K_{1,2}B_L(z) - R_0\left(K_{2,1}A_L(z) + K_{2,2}B_L(z)\right)z^{-2}} = \frac{\hat{B}(z)}{\hat{A}(z)}$$

Numerator polynomial:

$$\hat{B}(z) = (A_L(z) + B_L(z))\, z^{-(N+1)} \prod_{m=1}^{N}(1+k_m)$$

$$= \left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right) z^{-(N+1)} \prod_{m=1}^{N}(1+k_m)$$

$$\underbrace{\begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix}}_{\text{numerator coefficient vector}} = \mathbf{A}_L + \mathbf{B}_L$$

$$R_L(z) = \frac{B_L(z)}{A_L(z)} = -\frac{(b_L)_0 + (b_L)_1 z^{-1} + (b_L)_2 z^{-2}}{1 + (a_L)_1 z^{-1} + (a_L)_2 z^{-2}}$$

lip reflection coefficient vectors

$$\mathbf{B}_L = \begin{bmatrix} (b_L)_0 & (b_L)_1 & (b_L)_2 \end{bmatrix}$$

$$\mathbf{A}_L = \begin{bmatrix} (a_L)_0 & (a_L)_1 & (a_L)_2 \end{bmatrix}$$

Transfer Function (3): denominator polynomial

Transfer function (3):

$$\hat{H}_L(z) = \frac{(A_L(z) + B_L(z))z^{-(N+1)} \prod_{m=1}^{N}(1+k_m)}{K_{1,1}A_L(z) + K_{1,2}B_L(z) - R_0\left(K_{2,1}A_L(z) + K_{2,2}B_L(z)\right)z^{-2}} = \frac{\hat{B}(z)}{\hat{A}(z)}$$

Denominator polynomial:

$$\hat{A}(z) = K_{1,1}A_L(z) + K_{1,2}B_L(z) - R_0\left(K_{2,1}A_L(z) + K_{2,2}B_L(z)\right)z^{-2}$$

$$= \hat{a}_0 z^{-0} + \hat{a}_1 z^{-1} + \cdots + \hat{a}_{2(N+2)} z^{-2(N+2)}$$

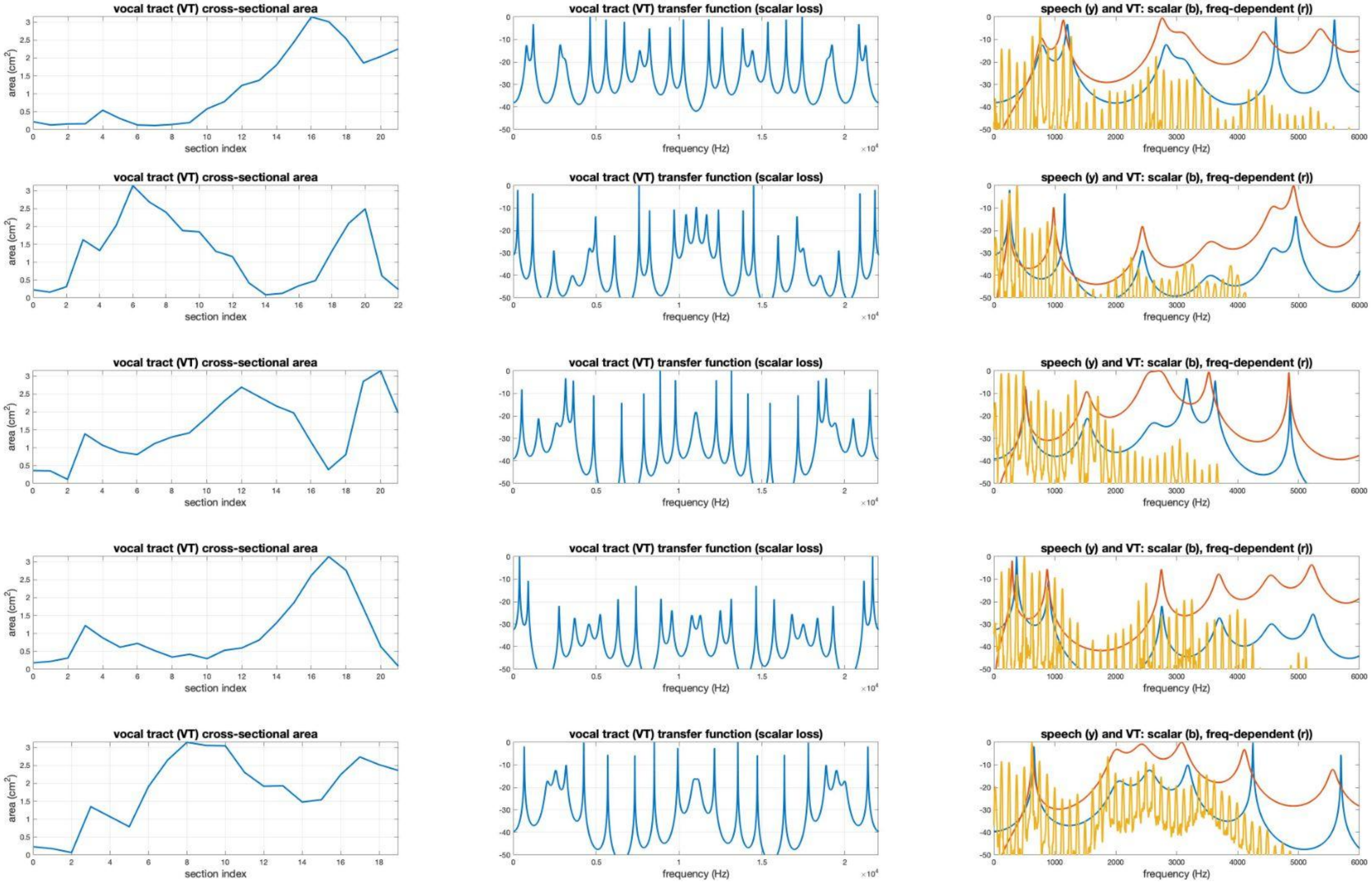
$$\left.\hat{\mathbf{A}}_N = \begin{bmatrix}\hat{a}_0 & \hat{a}_1 & \hat{a}_2 & \ldots & \hat{a}_{2(N+2)}\end{bmatrix}\right\}$$ denominator coefficient vector (with frequency-dependent lip reflection)

New boundary loss vector: $\hat{\mathbf{R}}_n = \underbrace{\begin{bmatrix}(a_L)_n & (b_L)_n & -(a_L)_n R_0 & -(b_L)_n R_0\end{bmatrix}^{\mathsf{T}}}_{\text{holds coefficients for } n^{\text{th}}\text{-order terms of } R_L(z)}$

Convolution of coefficients (matrix form):

$$\hat{\mathbf{A}}_N = \begin{bmatrix}\mathbf{C}_N \\ 0\,0\,0\,0 \\ 0\,0\,0\,0\end{bmatrix}\hat{\mathbf{R}}_0 + \begin{bmatrix}0\,0\,0\,0 \\ \mathbf{C}_N \\ 0\,0\,0\,0\end{bmatrix}\hat{\mathbf{R}}_1 + \begin{bmatrix}0\,0\,0\,0 \\ 0\,0\,0\,0 \\ \mathbf{C}_N\end{bmatrix}\hat{\mathbf{R}}_2,$$

$$\mathbf{C}_N = \begin{bmatrix}\mathbf{c}_N & 0 & 0 & 0 \\ 0 & \tilde{\mathbf{d}}_N & 0 & 0 \\ 0 & 0 & \mathbf{d}_N & 0 \\ 0 & 0 & 0 & \tilde{\mathbf{c}}_N\end{bmatrix}$$



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(thanks to Brad Story for audio and area functions)

Conclusions:

- Showed relationship between
  - piecewise-cylindrical waveguide model,
  - Kelly-Lochbaum scattering junctions reflection coefficients,
  - LPC.
- Showed how to incorporate a more accurate (higher-order, acoustically-informed) lip reflection filter into the vocal tract transfer function and feedback coefficients.

Future Work:

- Improved synthesis,
- Fit LPC coefficients to waveguide model,
- Improved inverse filtering,
- Estimation of $R_L(z)$ from LPC.