The Role of Lip Reflection/Transmission in the Relationship Between LPC and Waveguide Vocal Tract Models

Tamara Smyth[1] and Devansh Zurale

[1]trsmyth@ucsd.edu

Department of Music, UC San Diego, La Jolla CA, USA

October 17-20, 2021

WASPAA 2021

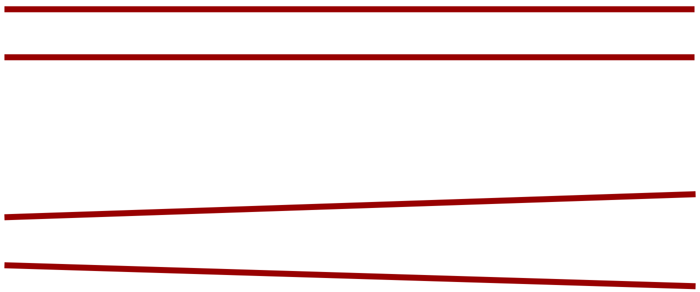
EEE Workshop on Applications of Signal Processing to Audio and Acoustics

UC San Diego

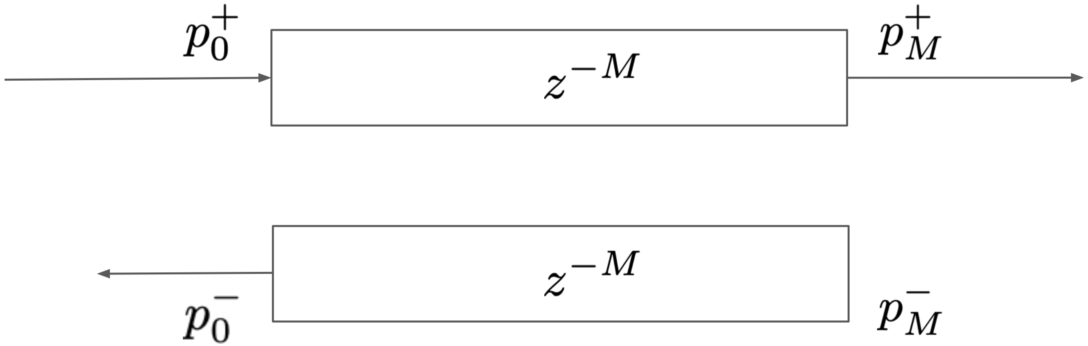
Summary

- Piecewise cylindrical waveguide model (Kelly-Lochbaum) of the vocal tract (acoustic tubes having varying cross-sectional area).
  - matrix representation leading to **three (3) transfer function**:
    - 1. non-polynomial form (sufficient for frequency-domain implementation)
    - 2. polynomial in z, with **scalar** loss
    - 3. polynomial in z, with **frequency-dependent** loss
  - acoustically-informed cylindrical open-end reflection function at the lips (shelf filter)
  - relationship to LPC
  - estimation of loss coefficients from LPC coefficients

Cylindrical and Conical Acoustic Tubes



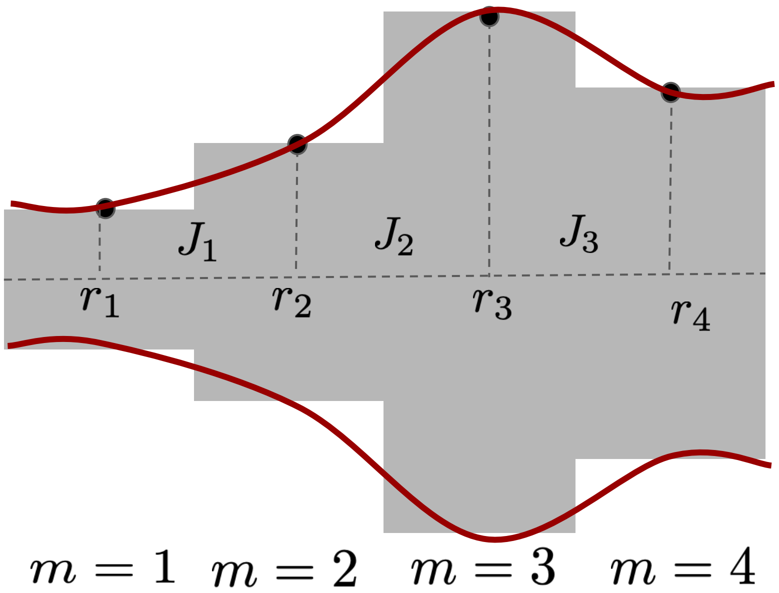
Waveguide Section



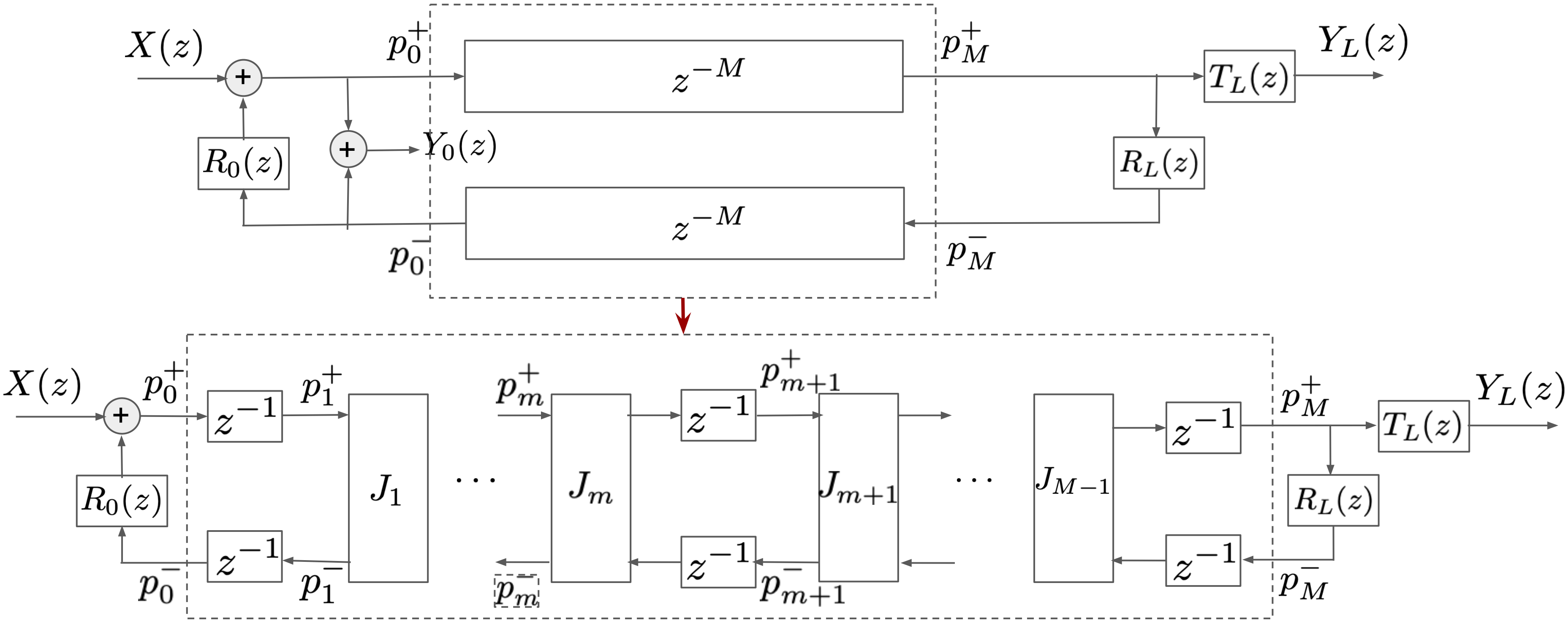
Acoustic Tube with Varying Cross-Sectional Area

Radii sampled/measured at regular spatial intervals

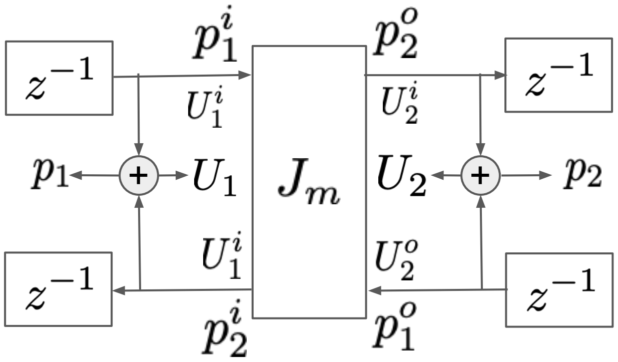
allows for a piecewise approximation



Piecewise Waveguide Model



Two-port scattering junction and port input/output (I/O)



***The law for conservation of mass and momentum*:**

- **pressure** at junction equals pressure on each port

$$p_J = p_1 = p_2 \longrightarrow p_1^i + p_1^o = p_2^i + p_2^o$$

- **volume velocity** on the each port sums to zeros

$$U_1 + U_2 = 0 \longrightarrow U_1^i + U_1^o = -\left(U_2^i + U_2^o\right)$$

Characteristic impedance (cylinders) **:** $Z_n = \dfrac{\rho c}{\underbrace{S_n}_{\text{cross-sectional area}}}$

$$U_n^i = \frac{p_n^i}{Z_n} \qquad U_n^0 = -\frac{p_n^o}{Z_n}$$

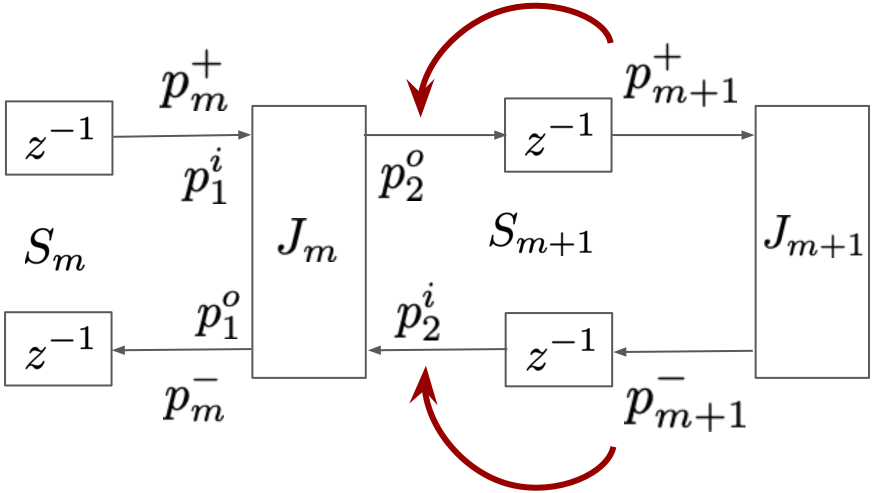
Matrix representation**:**

$$\mathbf{C}\begin{bmatrix} p_1^i \\ p_1^o \end{bmatrix} = \mathbf{D}\begin{bmatrix} p_2^i \\ p_2^o \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 1 \\ S_1 & -S_1 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 1 \\ -S_2 & S_2 \end{bmatrix}$$

Port I/O and Right/Left Traveling Waves



$$\mathbf{C}\begin{bmatrix}p_1^i\\ p_1^o\end{bmatrix} = \mathbf{D}\begin{bmatrix}p_2^i\\ p_2^o\end{bmatrix}$$

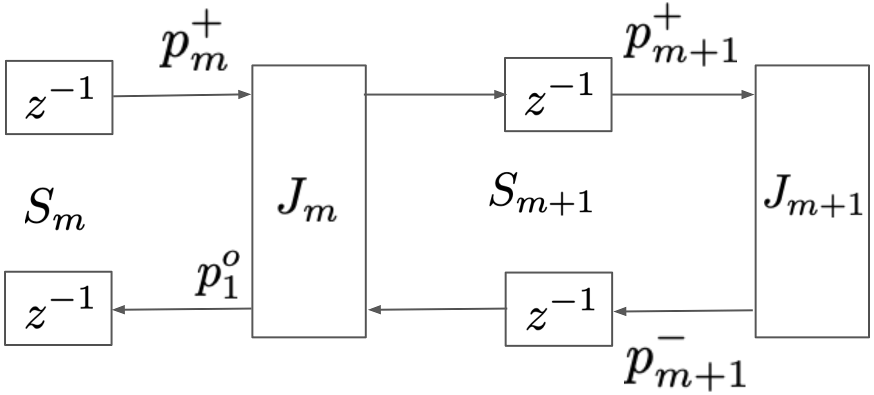
$$\underbrace{\underbrace{\begin{bmatrix}p_1^i\\ p_1^o\end{bmatrix}}_{\begin{bmatrix}p_m^+\\ p_m^-\end{bmatrix}}}_{\mathbf{p}_m} = \underbrace{\mathbf{C}^{-1}\mathbf{D}}_{(\mathbf{C}^{-1}\mathbf{D})_m} \underbrace{\underbrace{\begin{bmatrix}p_2^i\\ p_2^o\end{bmatrix}}_{\begin{bmatrix}p_{m+1}^- z^{-1}\\ p_{m+1}^+ z\end{bmatrix}}}_{\begin{bmatrix}0 & z^{-1}\\ z & 0\end{bmatrix}\underbrace{\begin{bmatrix}p_{m+1}^+\\ p_{m+1}^-\end{bmatrix}}_{\mathbf{p}_{m+1}}}$$

$$\mathbf{C}^{-1}\mathbf{D} = \frac{1}{2S_1}\begin{bmatrix}S_1 - S_2 & S_1 + S_2\\ S_1 + S_2 & S_1 - S_2\end{bmatrix}$$

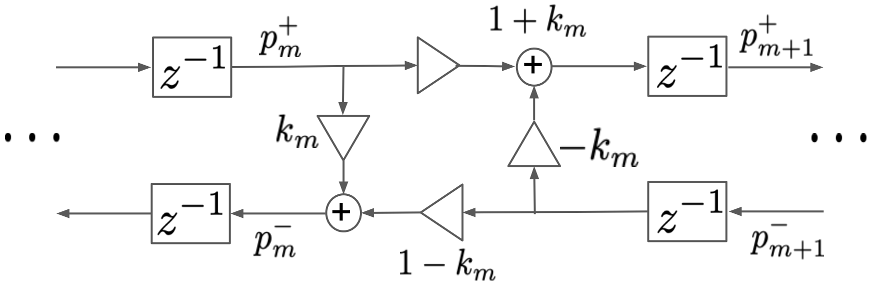
$$\mathbf{C}^{-1}\mathbf{D} = \frac{1}{2S_1}\begin{bmatrix}S_1 - S_2 & S_1 + S_2\\ S_1 + S_2 & S_1 - S_2\end{bmatrix}$$

$$(\mathbf{C}^{-1}\mathbf{D})_m = \frac{1}{2S_m}\begin{bmatrix}S_m - S_{m+1} & S_m + S_{m+1}\\ S_m + S_{m+1} & S_m - S_{m+1}\end{bmatrix}$$

Scattering Matrix and Reflection Coefficients



Kelly-Lochbaum Scattering Junction



$$\underbrace{\begin{bmatrix} p_m^+ \\ p_m^- \end{bmatrix}}_{\mathbf{p}_m} = \underbrace{\left(\mathbf{C}^{-1}\mathbf{D}\right)_m \begin{bmatrix} 0 & z^{-1} \\ z & 0 \end{bmatrix}}_{\mathbf{A}_m} \underbrace{\begin{bmatrix} p_{m+1}^+ \\ p_{m+1}^- \end{bmatrix}}_{\mathbf{p}_{m+1}}$$

Scattering matrix:

$$\mathbf{A}_m = \frac{1}{2S_m} \begin{bmatrix} S_m - S_{m+1} & S_m + S_{m+1} \\ S_m + S_{m+1} & S_m - S_{m+1} \end{bmatrix} \begin{bmatrix} 0 & z^{-1} \\ z & 0 \end{bmatrix}$$

$$= \frac{z}{1+k_m} \begin{bmatrix} 1 & k_m z^{-2} \\ k_m & z^{-2} \end{bmatrix}$$

where $\quad k_m = \dfrac{S_m - S_{m+1}}{S_m + S_{m+1}}$ (reflection coefficients)

“Chain” Scattering Matrix

Relationship between traveling waves in adjacent sections:

$$\mathbf{p}_m = \mathbf{A}_m \underbrace{\mathbf{p}_{m+1}}_{\mathbf{A}_{m+1}\underbrace{\mathbf{p}_{m+2}}_{\mathbf{A}_{m+2}\mathbf{p}_{m+3}}}$$

scattering matrix:

$$\mathbf{A}_m = \frac{z}{1+k_m}\begin{bmatrix} 1 & k_m z^{-2} \\ k_m & z^{-2} \end{bmatrix}$$

… and between first and final sections:

$$\mathbf{p}_1 = \underbrace{\left(\prod_{m=1}^{M-1} \mathbf{A}_m\right)}\mathbf{p}_M$$

$$\prod_{m=1}^{N} \mathbf{A}_m = \frac{z}{1+k_1}\begin{bmatrix} 1 & k_1 z^{-2} \\ k_1 & z^{-2} \end{bmatrix} \frac{z}{1+k_2}\begin{bmatrix} 1 & k_2 z^{-2} \\ k_2 & z^{-2} \end{bmatrix} \cdots \frac{z}{1+k_N}\begin{bmatrix} 1 & k_N z^{-2} \\ k_N & z^{-2} \end{bmatrix}$$

number of junctions:

$$N = M - 1$$

$$= \frac{\overbrace{z^N}^{\text{pure delay}}}{\underbrace{\prod_{m=1}^{N}(1+k_m)}_{\text{gain}}}\mathbf{K}_N$$

“chain” scattering matrix:

$$\mathbf{K}_N = \prod_{m=1}^{N}\begin{bmatrix} 1 & k_m z^{-2} \\ k_m & z^{-2} \end{bmatrix} = \begin{bmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \end{bmatrix}$$

Matrix Entries and Polynomial Coefficients

"Chain" Scattering Matrix:

$$\mathbf{K}_N = \prod_{m=1}^{N} \begin{bmatrix} 1 & k_m z^{-2} \\ k_m & z^{-2} \end{bmatrix} = \begin{bmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \end{bmatrix}$$

Entries are polynomial in z:

right column coefficients in reverse order

$$K_{1,1} = \sum_{m=0}^{N-1} c_{2m} z^{-2m} \qquad K_{1,2} = \sum_{m=1}^{N} d_{2(N-m)} z^{-2m}$$

$$K_{2,1} = \sum_{m=0}^{N-1} d_{2m} z^{-2m} \qquad K_{2,2} = \sum_{m=1}^{N} c_{2(N-m)} z^{-2m}$$

Initial coefficients: $c_0 = 1 \qquad d_0 = k_1$

Polynomial coefficient vectors have the form:

$$\mathbf{c}_N = \begin{bmatrix} c_0 \\ 0 \\ c_2 \\ 0 \\ \vdots \\ c_{2(N-1)} \\ 0 \end{bmatrix} \quad \mathbf{d}_N = \begin{bmatrix} d_0 \\ 0 \\ d_2 \\ 0 \\ \vdots \\ d_{2(N-1)} \\ 0 \end{bmatrix}$$

Coefficients are recursively defined:

$$\mathbf{c}_N = \begin{bmatrix} \mathbf{c}_{N-1} \\ 0 \\ 0 \end{bmatrix} + k_N \begin{bmatrix} 0 \\ \tilde{\mathbf{d}}_{N-1} \\ 0 \end{bmatrix} \quad \mathbf{d}_N = \begin{bmatrix} \mathbf{d}_{N-1} \\ 0 \\ 0 \end{bmatrix} + k_N \begin{bmatrix} 0 \\ \tilde{\mathbf{c}}_{N-1} \\ 0 \end{bmatrix}$$

~ denotes order of elements is reversed

Transfer Function 1: non-polynomial form

The transfer function is the (spectral) ratio of the model **output** to the **input**:

$$H_L(z) = \frac{Y_L(z)}{X(z)} = \frac{T_L(z)p_M^+}{p_1^+ z - R_0 p_1^- z^{-1}}$$

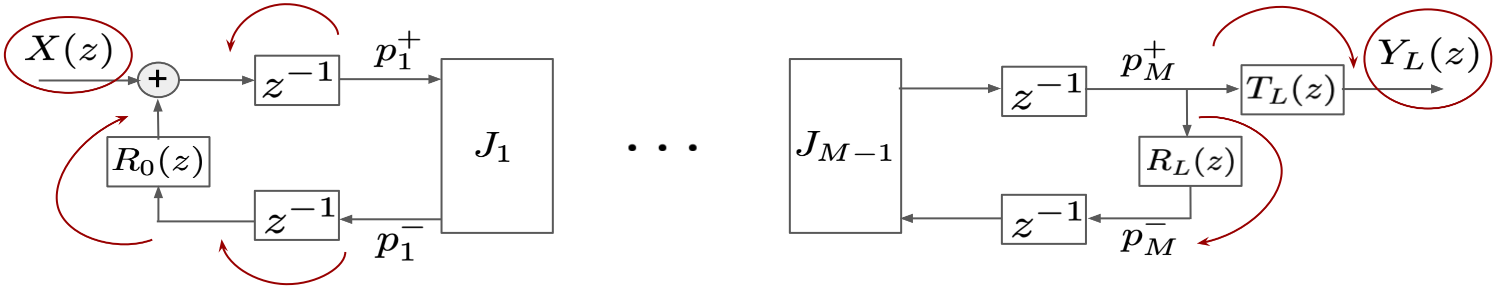
$$= \frac{T_L(z) z^{-(N+1)} \prod_{m=1}^{N}(1+k_m)}{K_{1,1} + K_{1,2}R_L(z) - R_0\left(K_{2,1} + K_{2,2}R_L(z)\right) z^{-2}}$$

Represent **first-section wave components** as a function of the **right-going component in the last section** using:

1. "chain" scattering matrix:

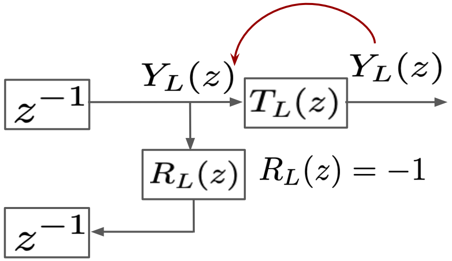
$$\mathbf{p}_1 = \frac{z^N}{\prod\limits_{m=1}^{N}(1+k_m)} \mathbf{K}_N \mathbf{p}_M$$

2. by definition: $p_M^- = p_M^+ R_L(z)$



Transfer Function 2: polynomial in z, scalar boundaries

$$H_L(z) = \frac{Y_L(z)}{X(z)} = \frac{\cancelto{1}{T_L(z)}\, z^{-(N+1)} \prod_{m=1}^{N} (1+k_m)}{\underbrace{K_{1,1} + K_{1,2}R_L(z) - R_0\left(K_{2,1}+K_{2,2}R_L(z)\right) z^{-2}}_{\text{lumped scalar loss}}} = \frac{B(z)}{A(z)}$$



$$A(z) = a_0 z^{-0} + a_1 z^{-1} + \cdots + a_{2(N+1)} z^{-2(N+1)}$$

Coefficient (column) vector:

$$\mathbf{A}_N = \begin{bmatrix} \mathbf{c}_N & 0 & 0 & 0 \\ \cdot & \tilde{\mathbf{d}}_N & 0 & 0 \\ \cdot & \cdot & \mathbf{d}_N & 0 \\ \cdot & \cdot & \cdot & \tilde{\mathbf{c}}_N \\ 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & \cdot \end{bmatrix} \mathbf{R} = \begin{bmatrix} \cancelto{1}{c_0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & \mathbf{0} & \mathbf{0} \\ c_2 & d_{2(N-1)} & d_0 & \mathbf{0} \\ 0 & 0 & 0 & 0 \\ c_4 & d_{2(N-2)} & d_2 & c_{2(N-1)} \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ c_{2(N-1)} & d_2 & d_{2(N-2)} & c_4 \\ 0 & 0 & 0 & 0 \\ \mathbf{0} & d_0 & d_{2(N-1)} & c_2 \\ \mathbf{0} & \mathbf{0} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cancelto{1}{c_0} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -R_0 \\ R_0 \end{bmatrix} \quad \mathbf{A}_N = \begin{bmatrix} 1 \\ 0 \\ a_2 \\ \vdots \\ 0 \\ a_{2N} \\ 0 \\ R_0 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 1 & \cancelto{-1}{R_L} & -R_0 & -\cancelto{R_0}{R_0 R_L} \end{bmatrix}^T$$

Relationship to LPC: impulse response of $H_L(z)$

Transfer Function:

$$H_L(z) = \frac{Y_L(z)}{X(z)} = \frac{z^{-(N+1)} \prod_{m=1}^{N} (1 + k_m)}{1 + \sum_{i=1}^{2(N+1)} \underbrace{a_i}_{\text{defined by } \mathbf{A}_N} z^{-i}}$$

Difference Equation:

$$y(n) = \prod_{m=1}^{N} (1 + k_m) \underbrace{x(n - (N+1))} - \sum_{i=1}^{2(N+1)} a_i y(n-i)$$

Input impulse:

$$x(n) = u(n) = 1, 0, 0, 0, \ldots$$

Impulse Response:

$$y(n) = h(n) = \begin{cases} 0, & \text{for } n < N+1 \\ \prod_{m=1}^{N} (1 + k_m), & \text{for } n = N+1 \\ -\sum_{i=1}^{2(N+1)} a_i h(n-i), & \text{for } n > N+1. \end{cases}$$

Relationship to LPC: impulse response of $H_L(z)$

Impulse Response:

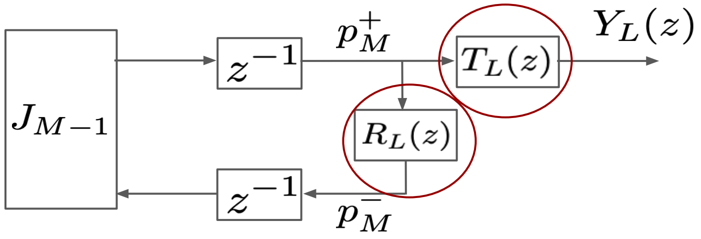
$$y(n) = h(n) = \begin{cases} 0, & \text{for } n < N+1 \\ \prod_{m=1}^{N}(1+k_m), & \text{for } n = N+1 \\ -\sum_{i=1}^{2(N+1)} a_i h(n-i), & \text{for } n > N+1. \end{cases}$$

Linear Prediction:

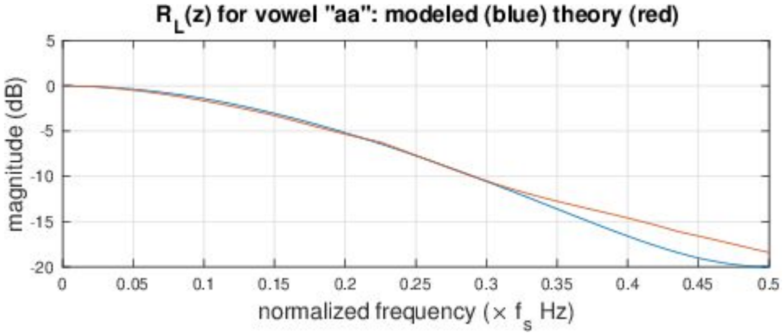
finds coefficients $\hat{a}_i$ such that $\underbrace{\hat{x}(n)}_{\text{predicted signal}} = -\sum_{i=1}^{\overbrace{p}^{\text{order}}} \hat{a}_i x(n-i)$ error: $\cancelto{0}{e(n)} = x(n) - \hat{x}(n)$

If order $p = 2(N+1)$ and $x(n) = h(n)$ then $\hat{x}(n) = h(n)$ and $\hat{a}_i = a_i$ $\mathbf{A}_N = \underbrace{\mathbf{A}_p}_{\text{LPC estimated coefficients vector}}$

Frequency-Dependent Lip Reflection



acoustically-informed reflection from an open cylinder:



Reflection function at the lips is made to be a **first-order shelf filter**:

$$R_L(z) = \frac{B_L(z)}{A_L(z)} = -\frac{(b_L)_0 + (b_L)_1 z^{-1}}{(a_L)_0 + (a_L)_1 z^{-1}}$$

coefficients are a function of the band-edge gain g_π (see paper)

When $0 < g_\pi < 1$, coefficients have the relationship: $(a_L)_0 - (b_L)_0 = -((a_L)_1 - (b_L)_1)$

Transmission function (amplitude complementary): $T_L(z) = 1 + R_L(z) = \dfrac{A_L(z) + B_L(z)}{A_L(z)}.$

Transfer Function 3: frequency-dependent lip reflection

Transfer function 1:

$$H_L(z) = \frac{Y_L(z)}{X(z)} = \frac{T_L(z) z^{-(N+1)} \prod_{m=1}^{N}(1+k_m)}{K_{1,1} + K_{1,2} R_L(z) - R_0 \left(K_{2,1} + K_{2,2} R_L(z)\right) z^{-2}} = \frac{B(z)}{A(z)}$$

Reflection function at the lips:

$$R_L(z) = \frac{B_L(z)}{A_L(z)} = -\frac{(b_L)_0 + (b_L)_1 z^{-1}}{(a_L)_0 + (a_L)_1 z^{-1}}$$

Amplitude-complementary transmission:

$$T_L(z) = 1 + R_L(z) = \frac{A_L(z) + B_L(z)}{A_L(z)}$$

Transfer function 3: frequency-dependent lip reflection:

$$\hat{H}_L(z) = \frac{(A_L(z) + B_L(z)) z^{-(N+1)} \prod_{m=1}^{N}(1+k_m)}{K_{1,1} A_L(z) + K_{1,2} B_L(z) - R_0 \left(K_{2,1} A_L(z) + K_{2,2} B_L(z)\right) z^{-2}} = \frac{\hat{B}(z)}{\hat{A}(z)}$$

Transfer Function 3: numerator polynomial

Transfer function 3:
$$\hat{H}_L(z) = \frac{(A_L(z)+B_L(z))z^{-(N+1)}\prod_{m=1}^{N}(1+k_m)}{K_{1,1}A_L(z)+K_{1,2}B_L(z)-R_0\left(K_{2,1}A_L(z)+K_{2,2}B_L(z)\right)z^{-2}} = \frac{\hat{B}(z)}{\hat{A}(z)}$$

Consider the sum of polynomials in the numerator:

$$\begin{aligned} A_L(z)+B_L(z) &= 1-(b_L)_0 + \underbrace{((a_L)_1-(b_L)_1)}_{-(1-(b_L)_0)}z^{-1} \\ &= (1-(b_L)_0)\underbrace{(1-z^{-1})}_{\text{simple first order high-pass filter}} \end{aligned}$$

shelf filter: when $0 < g_\pi < 1$

$$\overset{1}{\cancel{(a_L)_0}} - (b_L)_0 = -((a_L)_1-(b_L)_1)$$

The numerator becomes: $\hat{B}(z) = \underbrace{g}_{\text{gain}}\overbrace{(1-z^{-1})}\underbrace{z^{-(N+1)}}_{\text{pure delay}}$

gain: $g = (1-(b_L)_0)\prod_{m=1}^{N}(1+k_M)$

Transfer Function 3: denominator polynomial

Transfer function (3):

$$\hat{H}_L(z) = \frac{(A_L(z)+B_L(z))z^{-(N+1)}\prod_{m=1}^{N}(1+k_m)}{\underbrace{K_{1,1}A_L(z)+K_{1,2}B_L(z)-R_0\left(K_{2,1}A_L(z)+K_{2,2}B_L(z)\right)z^{-2}}_{\hat{A}(z)=\hat{a}_0 z^{-0}+\hat{a}_1 z^{-1}+\cdots+\hat{a}_{2N+3}z^{-2N+3}}} = \frac{\hat{B}(z)}{\hat{A}(z)}$$

Coefficient (column) vector:

$$\hat{\mathbf{A}}_N = \begin{bmatrix} \mathbf{c}_N & 0 & 0 & 0 \\ \cdot & \tilde{\mathbf{d}}_N & 0 & 0 \\ \cdot & \cdot & \mathbf{d}_N & 0 \\ \cdot & \cdot & \cdot & \tilde{\mathbf{c}}_N \\ 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & . \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{R}}_0 + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathbf{c}_N & 0 & 0 & 0 \\ \cdot & \tilde{\mathbf{d}}_N & 0 & 0 \\ \cdot & \cdot & \mathbf{d}_N & 0 \\ \cdot & \cdot & \cdot & \tilde{\mathbf{c}}_N \\ 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & . \end{bmatrix} \hat{\mathbf{R}}_1 = \begin{bmatrix} \hat{a}_0 \\ \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{a}_{2N+3} \end{bmatrix} \neq \mathbf{A}_N = \begin{bmatrix} 1 \\ 0 \\ a_2 \\ \vdots \\ 0 \\ a_{2N} \\ 0 \\ R_0 \end{bmatrix}$$

$$\hat{\mathbf{R}}_{0,1} = \begin{bmatrix} (a_L)_{0,1} & -(b_L)_{0,1} & -R_0(a_L)_{0,1} & R_0(b_L)_{0,1} \end{bmatrix}^T$$

Estimating Lip Reflection Coefficients $\hat{\mathbf{A}}_N$ (and LPC?)

First 4 elements of $\hat{\mathbf{A}}_N$:

$$\begin{bmatrix} \hat{a}_0 \\ \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = \begin{bmatrix} (a_L)_0 \cancelto{1}{c_0} & + & 0 & + & 0 \\ (a_L)_1 \cancelto{1}{c_0} & + & 0 & + & 0 \\ (a_L)_0 c_2 & - & (b_L)_0 d_{2(N-1)} & - & R_0 (a_L)_0 d_0 \\ (a_L)_1 c_2 & - & (b_L)_1 d_{2(N-1)} & - & R_0 (a_L)_1 d_0 \end{bmatrix}$$

Last 4 elements of $\hat{\mathbf{A}}_N$:

$$\begin{bmatrix} \hat{a}_{2N} \\ \hat{a}_{2N+1} \\ \hat{a}_{2N+2} \\ \hat{a}_{2N+3} \end{bmatrix} = \begin{bmatrix} -(b_L)_0 d_0 & - & R_0 (a_L)_0 d_{2(N-1)} & + & R_0 (b_L)_0 c_2 \\ -(b_L)_1 d_0 & - & R_0 (a_L)_1 d_{2(N-1)} & + & R_0 (b_L)_1 c_2 \\ 0 & + & 0 & + & R_0 (b_L)_0 \cancelto{1}{c_0} \\ 0 & + & 0 & + & R_0 (b_L)_1 \cancelto{1}{c_0} \end{bmatrix}$$

$$R_0 = -\frac{\hat{a}_{2N+3}}{(\hat{\mathbf{b}}_L)_2} = -\frac{\hat{a}_{2N+2}}{(\hat{\mathbf{b}}_L)_1}$$

elimination: $$\frac{\dfrac{\hat{a}_3}{\hat{a}_1} - \dfrac{\hat{a}_2}{\hat{a}_0}}{\dfrac{\hat{a}_{2N}}{\hat{a}_{2N+2}} - \dfrac{\hat{a}_{2N+1}}{\hat{a}_{2N+3}}} = \frac{(b_L)_0 (b_L)_1}{\cancelto{1}{(a_L)_1}\, \hat{a}_1} = D$$

$$(b_L)_0 = \frac{D \hat{a}_1}{(b_L)_1}$$

shelf filter: when $0 < g_\pi < 1$

$$\cancelto{1}{(a_L)_0} - (b_L)_0 = -\left(\cancelto{\hat{a}_1}{(a_L)_1} - (b_L)_1\right)$$

$$(b_L)_0 = 1 + \hat{a}_1 - (b_L)_1$$

quadratic equation: $(b_L)_1^2 - (1 + \hat{a}_1)(b_L)_1 + D\hat{a}_1 = 0$

$$(b_L)_1 = \frac{1 + \hat{a}_1}{2} \pm \sqrt{\left(\frac{1 + \hat{a}_1}{2}\right)^2 - D\hat{a}_1}$$

Summary

- Relationship between piecewise cylindrical waveguide model and LPC using **three (3) transfer functions**:
    - 1. non-polynomial form (sufficient for frequency-domain implementation)
    - 2. polynomial in z, with **scalar** loss
    - 3. polynomial in z, with **frequency-dependent** loss
        - acoustically-informed cylindrical open-end reflection function at the lips (shelf filter)
        - estimation of loss coefficients from LPC coefficients

- An accurate estimation of boundary losses may improve (with glottal pulse)
    - inverse modeling
    - estimation of reflection coefficients and cross-sectional area functions from LPC
    - fitting to a waveguide model (real-time parametric synthesis)