

Quantifying Pitch—Logarithms

Music 175: Pitch II

Tamara Smyth, trsmyth@ucsd.edu
Department of Music,
University of California, San Diego (UCSD)

June 4, 2019

- We have seen several times so far that *what we perceive is not necessarily the physical reality*.
- This is true with pitch: though connected to period and frequency of signals, it is not fully described by fundamental frequency.
- How do we **quantify** the perceptual attribute of pitch?
- Gustav Fechner (1873): *the perceived quality of a stimulus could be different from the way it is specified physically*.
 - perceived quantity is often a **logarithmic transformation** of the physical quantity.

1

Music 175: Pitch II

2

Just Noticeable Difference

- Just noticeable difference (JND): *minimum physical change detectable by a human observer* (see JND.pdf).
- Fechner:
 - used *just noticeable difference* (JND) in his experiments;
 - accumulated JNDs from the point where no change was detected up to a given stimulus level
 - number of JNDs used to express *the psychological level of a given stimulus*
 - found that the *first approximation* to his perceptually measured scales was a logarithmic scale.
- Doesn't yield quite the correct answer for pitch.

Direct Subjective Assessments

- S. Smith Stevens, countered this view saying that
 - the accuracy of discrimination of a stimulus (JND) isn't necessarily related to the overall perceived magnitude of a stimulus;
 - just as an accumulation of error in the position of a needle on a voltmeter doesn't necessarily yield voltage, an accumulation of JNDs doesn't necessarily have anything to do with perceived magnitude;
 - believed difference in stimuli should be measured by acquiring **direct subjective assessments**:
 - e.g., present a tone and inform subject that the tone is numbered 30 for loudness;
 - * if a subsequent tone seems twice as loud, subject should label it 60;
 - * if half as loud, label 15;

The Mel Scale

- Stevens (and others) wanted to construct a scale that reflected **how people hear musical tones**:
 - listeners were asked to adjust tones so that one tone was half as high as another, and other such subdivisions of the frequency range.
- Resulted in the Mel Scale (for *melody*):
 - a perceptual scale of pitches judged by listeners to be equal in distance from one another
 - the reference of 1000 mels was assigned as having a frequency of 1000 Hz (40 dB above threshold).
- A similar Bark Scale (Zwicker) ranges from 1 to 24 and corresponds to the first 24 critical bands.

Mel Vs. Hertz

- As frequency in Hertz increases, larger intervals are judged by listeners to produce equal increments.

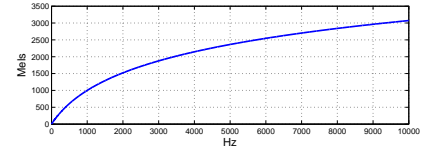


Figure 1: Mel scale vs. Hertz scale.

- Above 500 Hz, 2 octaves in Hz comprise about 1 octave in mels.
- Keyboard warped to match “equal” steps on the mel scale shows narrower keys at low frequencies.

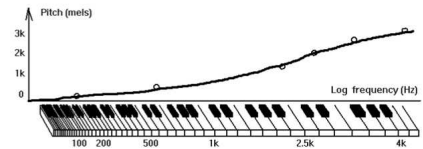


Figure 2: Warped keyboard depicting the mel scale.

Mel and Musical Intervals

- An actual keyboard is *isomorphic* to a log frequency scale (the physical distance of an octave is the same at high and low frequencies).

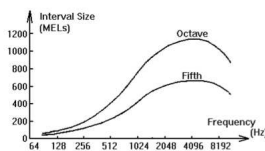


Figure 3: How musical interval sizes change across mel scale.

- The mel scale reflects that the perceived difference between notes decreases at the extreme ends of the keyboard.
- The mel scale DOES NOT reflect that there is a *constant* nature to musical intervals:
 - a minor third is that same interval no matter where it exists on the keyboard.
- This suggests 2 different kinds of pitch:
 - mel scale measurements
 - musical (melodic) pitch

Competence vs. Performance

- Equal steps on the mel scale do not sound *musically* equal:
 - a composition played using the mel scale may sound reasonable in the central range but increasingly incorrect toward higher and lower ranges;
 - Listen to wider spacing at the lower-end range of a **mel descending chromatic scale**
- Distinction can be seen in linguistics where Noam Chomsky distinguished between *competence* and *performance*:
 - grammar people know to be correct is different from what they actually use in speaking;
 - rules are known, but limited memory and processing power inhibit their use.
- Similarly, rules that govern musical interval relationships may be known, but limitations inhibit perception/production at extremes of the frequency range.

- The mel scale is not an appropriate musical scale, but can (and has likely) *inform(ed)* musicians:
 - musical compositions have statistically higher number of close intervals in the central frequency range.

- Visual space *IS* a morphophoric medium: a shape moved or rotated in space can still be recognized.

Pitch is a Morphophoric Medium

- Experiment (Attneave and Olson):
 - chimes playing G-E above-C below E (f=196, 330, 262 Hz)
 - first note would be transformed and subjects would adjust remaining notes until it sounded *correct*;
 - resulted not in *mel* but a *log frequency* scale within the common musical range, breaking down at high frequencies.
- There are many other attributes to the organization of sounds (timbre, spatial location, and loudness), but pitch and time have special importance;
 - they are *morphophoric*: capable of bearing forms that are preserved upon transformation;
 - pitch patterns (melodies and harmonies) can be moved up and down in pitch and still be recognized.
- Loudness, timbre and spatial locations are *NOT* morphophoric.

Dispensable/Indispensable Attributes

- Indispensable and dispensable attributes are akin to mediums that are, and are not, morphophoric.

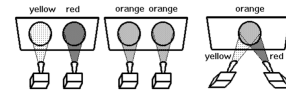


Figure 4: Two projected dots.

- Consider the following VISION example illustrating attributes of color and spatial position:
 1. two spots are projected side by side, one yellow and the other red;
 2. next, two spots are in same location but both orange (though dispensed with color, spatial position is preserved);
 3. next original red and yellow spots are projected on top of each other—only a single orange spot is visible;
- Spatial position is thus *indispensable*: by dispensing with position, original yellow and red dots are not distinguishable.

Indispensable and Morphophoric

- A similar AUDIO example with two loudspeakers:
 1. one loudspeaker plays middle C and the other middle E—one source is heard on the left, and one on the right;
 2. both pitches change to middle D—single source is heard located between two loudspeakers;

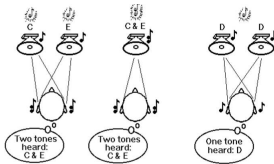


Figure 5: Two speakers carry two notes (left); one speaker carries two notes (center); two speakers carry the same note and there are no longer two notes (right).

- Pitch is thus *indispensable* and NOT analogous to color:
 - the analog to visual space is not auditory space, but auditory pitch;
 - visual space and auditory pitch are **morphophoric media**.

Musical Intervals

- Attneave and Olson:
 - believed an appropriate pitch representation could be found by examining subject’s ability to perceive/produce pitch patterns under transposition.
 - their representation corresponds to the log frequency scale, which, unlike mel scale, musical (and microtonal) intervals are preserved under transformations.
- Log scale does not represent the fact that octaves and perfect fifths, are *special*.
- Octaves and (to some degree) fifths are *culturally universal*:
 - though scales may differ culturally, they almost always contain an octave (and fifth) relationship;
 - tonal relationships of octaves show increased generalization over other intervals;

Spaces to Represent Pitch

- Play ascending 7ths on the the keyboard; it is possible to hear descending (chromatic) sequence;
- A helix on the surface of a cylinder places octaves immediately above and below each other.

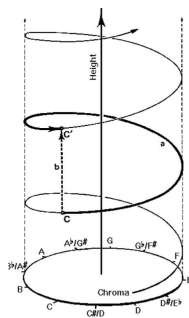


Figure 6: A nonlogarithmic pitch helix depicting octave equivalence.

- *Shepard tones*, ambiguous in height but unambiguous in pitch, can be made to spiral up the helix forever.
- **Chroma**: The position of a note within the octave; *chroma circle* is the base of the helix.
- **Pitch height** is the vertical position on the helix.

Chroma the basis of Pitch?

- Would a melody be recognizable if scrambled in terms of height while retaining chroma?
 - if the listener could attend only to chroma and ignore height, the answer would be “yes”.



Figure 7: Melody in randomized octaves (top), stretched (middle), normal (bottom).

- Researchers found that our perception of melody depends critically on height as well as chroma.
- A melody constructed with incorrect chroma but correct contour and height can be more recognizable than one that is completely scrambled in height.