Acoustics-like dynamics in signal-based synthesis through parameter mapping

Brendan Bernhardt Gaffney, Tamara Smyth
University of California, San Diego
bbgaffne,trsmyth@ucsd.edu

ABSTRACT

To ideally expand a sound synthesis parameter mapping strategy is to introduce complexity and capability without sacrificing its ease of use. Following work done with dynamical systems and catastrophe theory by René Thom, Sir E.C. Zeeman and others, we are able to create a general purpose model for introducing extended behaviors, akin to the dynamics of acoustic instruments, in low complexity interfaces without adding control parameters or losing the possibility of reverting to a simple, near-linear mapping.

Herein, we explore the principles of catastrophe theory, paying particular attention to the cusp model in which two input parameters yield a third and fourth describing the "catastrophic" events after which the theory is named. As acoustic systems possess several attributes of the catastrophic models, we experiment using the cusp model to enhance mapping of control parameters to FM synthesis parameters, in an attempt to give these signal-based virtual instruments the nuance and capability of their acoustic counterparts.

1. INTRODUCTION

The quality of a parametric sound synthesis model is not only determined by its produced sound, but also by the richness, depth, and intuitiveness of its control. As is the case with their acoustic counterparts, virtual musical instruments should engage users with music and sonic possibilities, allowing for exploration, discovery, and expression, with increased use, practice, and familiarity. A mapping strategy, therefore, may be evaluated by its "virtuosic ceiling" (potential for maturation with extended use) and its "entry fee" (ease of initial interaction) [1]. Balancing these two attributes is an important aspect in designing a system whereby performative gestures will be translated into synthesis parameters.

Physics-based synthesis models often have a myriad of possible synthesis parameters, offering possibilities in the produced sound akin to their acoustic counterparts. Though the complete set of possible parameters is usually too large to be effectively controlled by the user in realtime, there is usually a subset of "control parameters" that is naturally intuitive, largely because they are physical and relate to acoustic instruments with which the user has some familiarity and experience: blowing harder produces a louder sound; shortening the string produces a higher pitch. In addition to offering a low "entry fee" (ease of use) without requiring additional mapping, a quality physics-based model implements the dynamics of the system (the produced sound being dependent on both the current state of the model/parameters and their change over time), which also, by nature, offers possibilities that raise the "virtuosic ceiling" (maturation): blowing harder produces not only a louder sound, but also one that is brighter, harsher, detuned, or even the octave above (overblowing).

In signal-based models, the relationship between control and synthesis parameters is far less obvious (to both developer and user), and a mapping strategy is required to achieve a balance between ease of use and maturation. These mappings can be difficult to create, due to both their abstraction from a more obvious linear mapping, and their potential to create densely connected and difficult to debug and describe interactions. Existing strategies have incorporated generative methods to produce these mappings [2–4] and many have developed taxonomies to enable the decryption and development of these complex mappings [5, 6]. In this work we present an approach to parameter-mapping that, by borrowing concepts and models from catastrophe theory, aims to enrich signal-based models with the inherent complexities/intuitiveness of those that are based on some more natural, physically based musical interaction.

In an attempt to further the current mapping toolset, we have chosen to examine catastrophe theory as a potential set of theorems and models. Work done to extend the toolset available in creating these mappings is valuable to performer, composer and designer alike, as creating new primitives in mapping strategies yields a better set of design choices for the development of new mappings of control to synthesis, and therefore a more dynamic and nuanced interaction between instrument/interface designer, composer and performer.

In Section 2, we will examine catastrophe theory, its models and those attributes that indicate its potential value to parameter mapping development. In Section 3, we discuss its implementation, specifically in code via Pure Data and in a parameter mapping paradigm within frequency modulation synthesis. In Section 4 we discuss the results of these initial implementations, in Section 5 we examine the research to suggest possible topics for expansion and investigation, and in Section 6 we discuss the conclusions derived from our research.

2. CATASTROPHE THEORY

René Thom, a twentieth-century French mathematician, developed catastrophe theory as a means of explaining a set of complex singularities in geometry and mathematics. [7] [8] Thom's work inspired many to pursue the conclusions of catastrophe theory, not only in mathematics, but across disciplines. In his book *Catastrophe Theory*, Sir E.C. Zeeman, a British mathematician and champion of the relevancy of catastrophe theory across disciplines, presents several examples of simple, catastrophic systems outside mathematical fields [9]. A number of other researchers have used Thom's work in modeling a number of sociological [10], economic [11], physical [12], and biological [13] systems.

Catastrophe theory describes simple geometric models to explain systems that yield drastic changes in state in response to slowly changing attributes or parameters. These models have been developed from theorems proposed by Thom, that describe higher-dimensional geometry, specifically that of bifurcating sets of higher-order polynomials. His work concerned itself specifically with the discontinuities yielded by a number of special multi-dimensional geometric equations he termed *elementary catastrophes*, which are classified by the dimensions of their behavior and parameter spaces. The models Thom and Zeeman use to describe these systems are eloquent in that they are simple polynomials, whose real roots yield the stable states of the system, and whose coefficients shape the attributes of the thresholds and surface of the models [14].

While these previous implementations of catastrophe theory have little to prove for our mapping here, they point to the validity of catastrophe theory models in a range of applications and disciplines.

The elementary catastrophe we will concern ourselves with herein will be a lower dimensional model, due to its potential for representation on paper and its relative ease of comprehension and application. The model is the cusp catastrophe, which is described by a simple cubic polynomial, and from a two dimensional control space yields a third, potentially bimodally distributed behavior axis, whose value is dependent on previous states and trajectory through our control space. The cusp is manipulated by adjusting the coefficients of a polynomial, using two of these coefficients as navigational axes of a control space.

2.1 The Cusp Catastrophe

Catastrophe theory comprises a number of models that relate or map "attributes" to "behavioral" states. One such model, called the cusp catastrophe, is given by

$$c_h x^3 + b c_w x + a = 0, \tag{1}$$

where c_h and c_w are used to change the cusp height and width, respectively, and coefficients a and b are input control parameters. The surface C in Figure 1 is the control surface created by axes a and b, while the *manifold cusped* surface M (above C) is defined by the *real* roots of (1). The positive and negative values of x create the two *sheets* (upper and lower regions) of M.

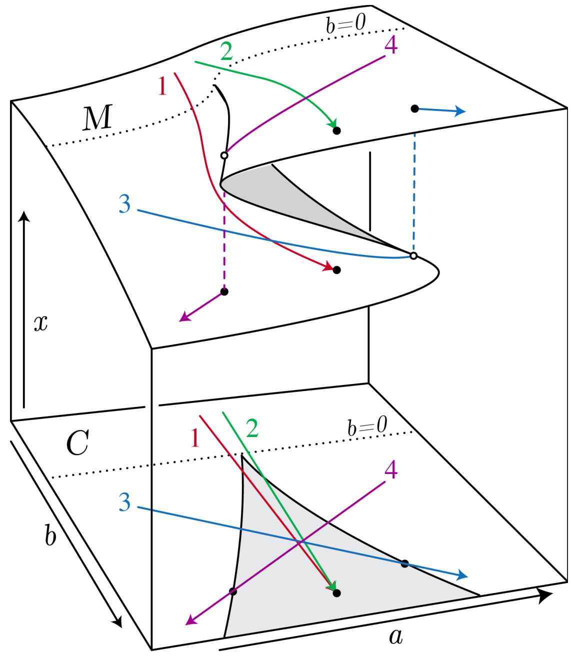

Figure 1. The elementary cusp catastrophe. Our variable axes b (splitting factor) and a (normal factor) and behavior axis x are labeled in the control surface C, and several trajectories through this control surface are traced both on C and their resulting values for x are traced on the behavior manifold M.

Since (1) is a cubic polynomial, it has three roots. The shaded area on control surface C, indicates values of a and b for which all three of these roots are real—*the bifurcating set*. These three real roots define the folded or "cusped" region of the manifold surface M. Outside the shaded region in C lie values for a and b yielding only a single real value for x. The two curved lines outlining the shaded area are thus thresholds for which a and b yield single or multiple (bifurcating) values of x. Bifurcating values of x appear for values of $b > 0$. For $b < 0$, x increases continuously with a. Static coefficients, c_h and c_w, effectively scale the coefficients a and b, thus skewing the dimensions of the cusp.

Fig. 1 shows several trajectories, labeled 1-4, of linearly changing values for a and b. Trajectories 1 and 2 on C, which originate on either side of the bifurcating set, produce different values for x, shown by corresponding trajectories 1 and 2 on M, despite a common destination point and similarly changing values of a and b. This exemplifies the first of the catastrophe model's attributes:

Attribute 1. *The behavior resulting from a given set of control values is dependent both on initial conditions and previous behavior.*

Trajectories 3 and 4 illustrate the characteristic jumps, or "catastrophes," after which the models are named, which occur when moving from the bifurcating set to the non-bifurcating set (jumps are illustrated in Fig. 1 using dashed lines on M and occur at points on C when the trajectory moves from inside to outside the shaded area). Furthermore, if a trajectory exits across the same threshold from

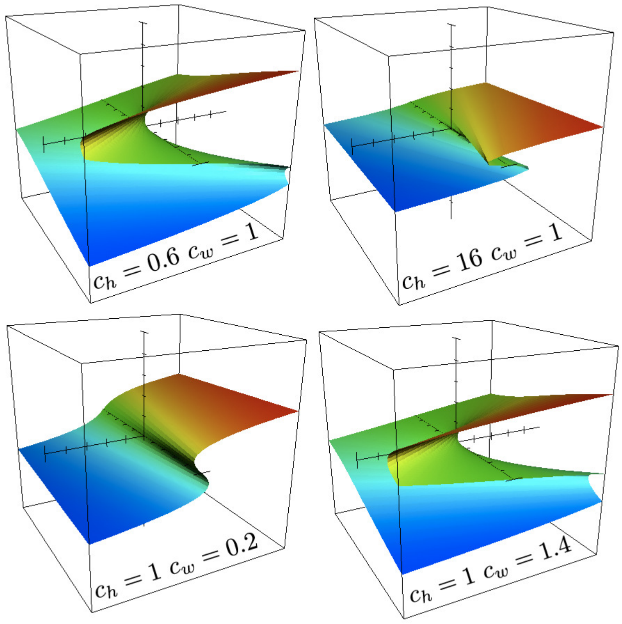

Figure 2. The effects of different values of c_h and c_w on the shape of our cusped surface. Axes are the same scale in all four plots.

which it entered, i.e. remains on the same "sheet", no jump occurs. This exemplifies the second of the model's attributes:

Attribute 2. *Jumps in the value of x occur only upon exiting the bifurcating set onto a new sheet.*

2.2 Applying to Dynamic Systems

Though the cusp model has two input parameters, it generates another two, yielding a total of four possible synthesis/application parameters: a, b, location x on the cusp manifold surface, and a binary value indicating whether x is on the upper or lower sheet. This increase indicates a potential value in parameter mapping, as it suggests a possible mapping of a simple control space to a more complex dynamic or sound synthesis system.

Any system that exhibits:

1. bimodal distributions of behavior for a dynamic input (relating to Attribute 1),
2. drastic changes in behavior despite slowly changing control parameters (relating to Attribute 2).

is a potential candidate for representation by a catastrophe model. Several such systems exist in music applications. In particular, blowing into the mouthpiece of a saxophone presents an example of an acoustic system that exhibits these two attributes: slowly varying embouchure and blowing pressure (corresponding to control axes a and b) for a given fingering, produces a sound that can leap in register/octave—a bimodality in state (Attribute 1) that can result in a jump in x (Attribute 2). That is, the tendency of the horn to lock into an upper or lower register, based on its previous state, exhibits Attribute 1. The tendency for a horn to jump *catastrophically* in register despite slow changes in control exhibits Attribute 2.

This simple catastrophic model of the saxophone shows the natural and musical behavior of control parameters fed through a cusp model. This nuanced behavior, coupled with the simplicity of the mathematics and rules of behavior, point to a potentially rewarding mapping strategy.

3. IMPLEMENTATION

In implementing catastrophe theory and polynomial equations in a mapping strategy, we are looking for complexity and capability in expression without diminishing the ability to use an interface effectively and easily. Furthermore, we hope to reward maturation with an interface, providing a more complex and nuanced interaction with the interface over time, more so than previously available without a complex mapping. The cusp model (1) is implemented as a Pd external object (written in C) [15], which offers a real-time interactive programming environment popular among computer musicians. Several patches from our experimentation, and the `cusp~` external, are available for download [16].

The first step in implementation is to fully understand the effects of manipulating the coefficients of (1). Initial tests were run in graphing programs to illustrate the width and height of the cusp for different values of c_h and c_w (see Figure 2). Following this, implementation is straightforward. First, the Cusp model is coded as a function having four input parameters, two static (c_w and c_h) and two dynamic (a and b), and two returned values, x and a binary indicating on which sheet, HIGH or LOW, x lies. Through experimentation, c_h was deemed unnecessary as it was *nearly* a scaling of x that could instead be more effectively and predictably applied as a linear scaling of the output (reducing required inlets in the Pd external to three).

The function uses the cubic polynomial solver in the GNU Scientific Library, as it returns only real values (and not complex values that have nothing to do with surface M). This function takes our three coefficients above and three pointers to memory locations in which it stores the returned roots of our equation. It also returns an integer indicating whether there is one or three real roots, effectively indicating whether we are in a bifurcating or non-bifurcating set of values for a and b.

Finally, a state variable is used to "remember" on which sheet of the cusp surface x resided in the previous time step, determining which of the roots of x, lower or upper, should be returned (the middle value is not considered in these models). Therefore, in this example we have a doubling of possible control parameters: the original a and b, plus two more given by the cusp model, x and sheet of x.

This code can be further optimized by implementing our own polynomial solver instead of calling an outside function (which itself makes several outside function calls). Furthermore, a number of other techniques can be used to determine the correct root, and some of these may be more optimal. Because this code, wrapped as a Pd external, is computed for every sample, it may be used in waveshaping and audio-rate modulation, as well as control rate paradigm.

4. APPLICATION AND RESULTS

Here, we choose to explore its use in the context of an FM (frequency modulation) synthesizer, to see how acoustic behavior as described in Section 2.2 can be incorporated in a signal-based model. A very simple implementation can be observed in Figure 3, where the index of modulation is controlled by both x and the binary HIGH/LOW sheet variable, while the carrier and modulator frequency are controlled by a and b, respectively.

The patch illustrated in Fig. 3 was used as an experimentation platform for determining the effect of our two generated parameters in very minimal signal-based synthesis system. Frequency modulation was chosen for our familiarity with its common control mappings and produced sound.

The interface chosen for initial experimentation was a touch sensitive trackpad, which returned an x and y value for a finger moved about its surface. By implementing our mapping with the cusp modeling, we essentially are able to traverse the lower and upper sheets of the model with our finger, and dictate the behavior based on our trajectory across and around the thresholds of the model, much as the paths in Fig. 1. This allows nuanced control of the output values, as it is immediately possible for a novice user to locate, empirically, the location of these thresholds and quickly learn to exploit or avoid their happening.

4.1 Cusps in Timbre, Amplitude and Pitch Control Paradigms

The cusp in the above patch maps timbre to our cusp model and pitch to our input a and b. Several other implementations were made systematically to determine by isolation the effect of cusp models on signal-based synthesis's most often used parameters, timbre, amplitude and pitch.

In Fig. 3 we have mapped our FM timbral parameter, the index of modulation, to the x output by the cusp model. We also tested this same system without the changing pitch, and therefore isolated timbral control with the cusped model. This yields an interesting, pseudo-vocal behavior, jumps in sideband presence and spread affecting a dynamic, albeit it expressively limited, control of timbre.

In other experiments, amplitude and pitch were controlled with the new complex yielded parameters x and sheet of x. An interesting result of this experimentation was the effect of the changing x without leaving the current sheet. The effect was to obtain a vernier control of a small subset of the accessible control space, effectively enabling a magnification of the values of x available on a given sheet. When mapping to amplitude, at higher values of b, where the sheets are most distant and the values of x therefore more disparate, this amounts to an ability to make nuanced changes in loudness at either a lower *piano* dynamic or, after jumping sheets, fine adjustments at a higher *forte* dynamic.

It is in our mapping of this model to pitch that the aforementioned "magnification" of certain subsets of the control space is most notable. The lower portion of the control space allowed minute control of a lower pitch subset, and after a jump, minute control of a higher pitch subset. As the middle pitches can be accessed by simply decreasing b, this introduces a very interesting paradigm of control. A scale running from lowest to highest pitch sets therefore runs in a horseshoe shape, retreating around the bifurcating set of values of a and b and back out to the higher sheet, without encountering catastrophic jumps but increasing the nuance of control at all points. Furthermore, jumps of different sizes between registers can be made easily and with some precision by simply locating the proper crossing point of the threshold to take a path through.

These mappings to signal-based synthesis parameter primitives helped illustrate the value of these models to the expansion of available parameter mapping strategies. To our initial goal of introducing acoustic-like behavior to these simpler signal based models, it points to observed behaviors, like the selective magnification, that may map to acoustic-like paradigms.

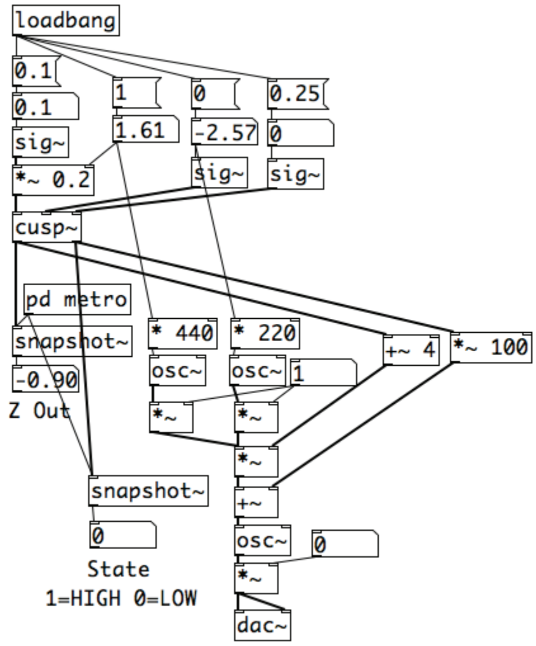

Figure 3. A simple patch illustrating the mapping of our four synthesis parameters (two generated by our cusp model, two direct from control) to a simple FM-based synthesis algorithm.

4.2 Introducing Acoustic-like Behavior to a Signal Model

The main purpose of these experiments is to determine if the two additional parameters generated by the cusp model, x and sheet of x, are useful and intuitive synthesis parameters. As previously illustrated, it can be shown that acoustic systems have a tendency to behave like the cusped model, so our aim was to investigate the presence of some more natural or acoustic-like behavior in the mapping.

We can show this behavior by observing Attributes 1 and 2 in the process of using the interface, and determine if they are related as predictable and controllable features to a user.

By default a simple FM synthesis model has no inherent acoustic-like qualities, as FM linearly mapped to the control parameters of a trackpad or other continuous controller is dissimilar from any existing acoustic system. This allows us to track the effect of introducing the cusp mapping, and evaluate it independently of the synthesis algorithm's behavior. This isolation of a mapping is key to evaluating its worth, as many synthesis algorithms behave naturally and effectively without an intermediate mapping between control and synthesis parameters.

First, the implementation of this model effectively enlarges the parameter space of our sound synthesis system, as Attribute 1 shows that a large portion of our control surface has two possible values of x. By introducing this bifurcating behavior, like that found in acoustic systems, the parameter space of our interface widens, and therefore a larger portion of the sound space of the synthesis algorithm is available to a performer.

Furthermore, by using cusp-generated parameters x and sheet of x to control the index of modulation, an abstract synthesis parameter without an acoustic analog, we introduced a way of jumping between timbres of the synthesis algorithm. Each sheet of the cusp maps to two different sound spaces, with finer adjustments accessible using x, and a user can switch purposefully from one to another. These jumps, described by Attribute 2, introduce a triggered, more dynamic behavior to our previously linear interface.

Also, the complex behaviors of wind instruments discussed in Section 2.2 can be modeled with careful application of the cusp model in mapping control parameters. In experimentation, the sheet of x was mapped to pitch, while x was mapped to index of modulation. This mapping closely resembles the articulation of a single keying of the saxophone, where an increase in embouchre and blowing pressure will push the horn to both jump in octave (a catastrophic jump in pitch) and harshen in timbre (an accompanying increase in x).

The selective magnification also has many acoustic analogs. Jumps between registers as desribed above, with some small, more nuanced adjustments available on either end of these jumps, also closely resembles paradigms present in wind instruments. Again, the jumps can be associated with pitch, but if mapped with proper scaling of x instead of the binary sheet of x, small adjustments in intonation can be made in each register with some precision.

By identifying and exploiting these acoustic behaviors in our new mapping, which introduce more complex expression and control in an otherwise simple system, we have increased the potential for engagement and discovery in the process of learning a musical interaction with a digital system. We have done this by relating the interaction with a digital musical instrument to interactions a performer and composer are more likely to have some experience with. Furthermore, we have helped mediate the potential expressiveness of the vast sound space available in signal-based models to a much smaller and simpler control space.

4.3 Balancing Complexity and Cost

Another main focus of these experiments is to determine the ability for a user to easily acquire the mapping and behavior of the interface, and if maturation with the system is rewarding over time. While the initial experiments are basic, results indicate that the mapping has the potential to fulfill our two desired features of a new mapping, namely the low entry fee and high virtuosic ceiling.

By scaling and offsetting our input parameters, the bottom half of the trackpad can be kept near-linear, or without bifurcation (by keeping $b < 0$, as shown in Fig. 1), and therefore more immediately intuitive, while still allowing the top half to exhibit the more complex bifurcating behavior. By building this duality into the interface, it is possible for a simple interface to yield both easily accessible and more complex behaviors.

Furthermore, several cusps can be implemented with differing dimensions and locales on the control surface by simply adding more of these models in the intermediary mapping layer. These additional mappings afford the same designed duality in simplicity and complexity. We can therefore introduce the complexity in behavior available with several cusps without accumulating complexity in the lower half of our mapping and eliminating its ease of acquisition. These two conditions satisfy the desire to find mappings both easy to acquire and rich in complexity and nuance that can be acquired over time.

5. CONCLUSIONS

Catastrophe theory, as laid out by Thom and others, allows us the means by which to extend the currently available tools used in parameter mapping. It does so by supplying models in which a low number of parameters yield new and complex output behaviors.

The cusp model from catastrophe theory is ideal for several reasons. First, it is relatively easy to understand, due to its ease of representation in three dimensions on paper, and its low order polynomial description. It is also easy to implement in code, and easier still to include in a mapping strategy once encapsulated into an external or its equivalent outside of Pd.

The likeness of the cusp model to acoustical systems further extends the implemented mappings, by extending their behaviors, size of control space, and introducing control that is intuitive and nuanced like that of an acoustical system. It allows for an acoustics-like dynamic for a signal based synthesis algorithm, by introducing an intermediary mapping layer. We can see the real mapping benefits of introducing both dynamic jumps in parameter range and the effect of bifurcating control surfaces in both ease of control and likeness to acoustic analogs.

Furthermore, the cusp model, and other polynomials like it, is possible to implement in a non-complicating manner. It can be subtle or drastic, with or without linear mapping possibilities behind some threshold, and multiplied in number, all potentially without cost to the initial acquisition of the interface's function and the ease that simple mappings allow novice users to begin making sound in a purposeful manner. It is this introduction of complexity without cost that highlights the possibilities of these equations as tools in the mapping strategies of larger, more complex algorithms.

6. FURTHER RESEARCH

While this paper focuses on a simple implementation of a lower complexity model of catastrophe theory, there is still more to do in terms of applying these models and evaluating their wider uses and conclusions.

First and foremost, examination of all of catastrophe theory's models, not simply those more conveniently laid out on paper, is called for. While they do not guarantee the possibility of nuanced or simple behavior like the cusp catastrophe does, their higher level of input and output parameters suggest their potential relevancy. One such model, the butterfly model, is suited for further research, as its surface can also be traced with two parameters, and its coefficients and behaviors are more complex. Initial experimentation with the butterfly model's behaviors show some promise for parameter mapping.

Second, catastrophe theory itself may well be worth examining in music and sound synthesis outside of parameter mapping. Its relevancy in physics to describe complex behaviors resembling resonance point to its potential use in modeling the behavior of musical instruments, in terms of musical information retrieval or parameter estimation techniques.

In effect, catastrophe theory's implementation herein has only been the initial stages of applying a theory to a new discipline. The scope of this article was necessarily smaller in scope to more carefully explore a single implementation, and without expansion outwards, this topic is not fully explored or tested.

7. REFERENCES

[1] M. Wanderley and N. Orio, "Evaluation of input devices for musical expression: Borrowing tools from hci," *Computer Music Journal*, vol. 26, no. 3, pp. 62–76, 2002.

[2] C. G. Johnson, "Exploring the sound-space of synthesis algorithms using interactive genetic algorithms." *Interface*, 1999.

[3] P. Dahlstedt, "Free Flight in Parameter Space: A Dynamic Mapping Strategy for Expressive Free Improvisation," *Applications of Evolutionary Computing*, 2008.

[4] ——, "Creating and exploring huge parameter spaces: Interactive evolution as a tool for sound generation," in *Proceedings of the 2001 International Computer Music Conference*, 2001, pp. 235–242.

[5] A. Hunt, M. M. Wanderley, and R. Kirk, "Towards a Model for Instrumental Mapping in Expert Musical Interaction," *Proceedings of the 2000 International Computer Music Conference*, pp. 209–212, 2000.

[6] A. Hunt, M. M. Wanderley, and M. Paradis, "The Importance of Parameter Mapping in Electronic Instrument Design," *Journal of New Music Research*, vol. 32, no. 4, pp. 429–440, Dec. 2003.

[7] R. Thom, *Structural stability and morphogenesis*. Addison Wesley Publishing Company, 1989.

[8] ——, "Structural Stability, Catastrophe Theory, and Applied Mathematics," *SIAM review*, vol. 19, no. 2, pp. 189–201, 1977.

[9] E. Zeeman, *Catastrophe theory, Selected papers 1972-1977*. Addison Wesley Publishing Company, 1977.

[10] A. Wilson, *Catastrophe Theory and Bifurcation (Routledge Revivals): Applications to Urban and Regional Systems*. Routledge, 2012.

[11] H. R. Varian, "Catastrophe theory and the business cycle," *Economic Inquiry*, vol. 17, no. 1, pp. 14–28, 1979.

[12] T. B. Benjamin, "Bifurcation phenomena in steady flows of a viscous fluid. i. theory," *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, vol. 359, no. 1696, pp. 1–26, 1978.

[13] L. Cobb and S. Zacks, "Applications of catastrophe theory for statistical modeling in the biosciences," *Journal of the American Statistical Association*, vol. 80, no. 392, pp. 793–802, 1985.

[14] P. T. Saunders, *An introduction to catastrophe theory*. Cambridge University Press, 1980.

[15] M. Puckette *et al.*, "Pure data: another integrated computer music environment," *Proceedings of the Second Intercollege Computer Music Concerts*, pp. 37–41, 1996.

[16] (2013, April). [Online]. Available: http://www.burnheartsynth.com/files/cusp_tilde.zip