INTRODUCTION

Blowing into the mouthpiece of a saxophone allows the player to control the oscillation of the reed by creating a pressure difference across its surface. When the reed oscillates, it creates an alternating opening and closure to the bore, resulting in a periodic train of pressure pulses, or a *reed pulse* sequence [1], that enters into the instrument bore.

The propagation of pressure waves in the cylindrical and predominantly conical sections of the saxophone may be modeled in one dimension using a digital waveguide, with a bi-directional delay line accounting for the acoustic propagation delay, and additional filter elements accounting for losses distributed along the length of the instrument and at the boundaries. Though open tone holes, used by the player to control sounding pitch, do complicate the issue, they can still be modeled within a one-dimensional waveguide context by *lumping* their effects at waveguide boundaries (i.e. with the bell).

For wind instruments, one of the primary ways in which a performer controls sound production, aside from changing pitch using instrument tone holes/keys, is by changing the air flow into the bore through alterations of blowing pressure and embouchure. In saxophone playing, estimation of the signal generated by the reed, the *reed pulse*, by inverse filtering the effects of the instrument can, therefore, yield a signal holding many of the more subtle playing parameters.

In [2] a parametric waveguide model of a saxophone *without tone holes* is presented, where the bore resonance may be set with a pure delay, and where boundary reflection and transmission filters, obtained using a developed measurement and post processing technique, are used to account for the acoustic behaviour of the bell. Here, an extension of this work, a model of a saxophone *with tone holes* is developed from waveguide theory and acoustic measurement, making it more suitable for *inverse* applications.

INVERSE FILTER

Ignoring the time-varying component in the reed/mouthpiece reflection, the saxophone response Y_B to input pressure $X = Z_0 U$, where U is the volume flow and Z_0 is the characteristic wave impedance, may be expressed in the z domain as

$$Y_B(z) = X(z)H(z), \tag{1}$$

where $H(z)$ is the saxophone reed pulse transfer function. If wave propagation in the instrument is modeled as a 1-D waveguide, with reflection at the mouthpiece R_M, bell reflection and transmission (along with lumped effects of open tone holes) functions R_B and T_B, respectively, and propagation loss λ, transfer function H can be expressed in the z domain as

$$H(z) = \frac{z^{-L}\lambda(z)T_B(z)}{1 - z^{-2L}\lambda^2(z)R_M(z)R_B(z)}, \tag{2}$$

where z^{-L} simulates the propagation delay in the purely conical section of the bore. The reed pulse sequence $X(z)$ may then be obtained by inverse filtering,

$$X(z) = G(z)Y_B(z) \tag{3}$$

where the inverse filter is given by

$$G(z) = \frac{1}{H(z)} = \frac{1 - z^{-2M}\lambda^2(z)R_M(z)R_B(z)}{z^{-M}\lambda(z)T_B(z)}. \tag{4}$$

The inverse problem of estimating the reed pulse X, therefore, reduces to the problem of estimating the filter G, so that it may be applied to the sound produced by the instrument Y_B, to reveal a signal expected to hold more *control* information than one clouded by the effects of the *instrument*.

The real difficulty arises in that the saxophone instrument transfer function H, and thus its inverse G, is expected to change during performance. As the perform applies various fingerings to change the instrument's pitch, the changing configuration of open and closed tone holes will produce different patterns of reflection and transmission along the bore length, effectively changing the functions R_B and T_B, into which these effects are lumped.

Though one might use a more elaborate waveguide model, with scattering junctions to model each of the instrument tone holes, the problem of knowing the state of each tone hole (open or closed) from the saxophone signal Y_B still remains. Instead, therefore, an established measurement and post processing technique is further developed to obtain the saxophone transfer function H for every possible fingering used when controlling the B-flat tenor saxophone. Statistical methods may be used on the recorded saxophone signal Y_B to determine the most likely transfer function H, and then the corresponding inverse filter G may be applied.

MEASUREMENT AND ESTIMATION OF INSTRUMENT TRANSFER FUNCTION

The original measurement technique being employed here, fully described in [3], allows for estimation of wind instrument waveguide elements from incremental measurements of a system impulse response. The measurement system consists of a 2-meter long tube with speaker and co-located microphone at one end for introducing a driving signal and simultaneously measuring the response at the same position $Y_0(z)$ (the response being a measurement of pressure, but corresponding strongly to the input impedance). A second microphone is placed outside the tube, 7 cm from the opposite end, on axis with the center of the tube. The system is first measured with the opposite end closed and then with an appended device under test (DUT).

When the DUT is sufficiently short as compared to the measurement tube (e.g. wind instrument bells), the measured system impulse response consists of a sequence of evenly spaced echos, sufficiently spaced to allow for isolation without loss of information. As shown in [3], windowed echos may be transformed, combined and manipulated algebraically to yield estimates of DUT reflection and transmission frequency responses, for use in the context of a waveguide model.

When the DUT is longer, however, such as when it is a complete saxophone with *both* bore and bell, individual echos from the system impulse response are smeared in time and can no longer be easily windowed. For this reason, the post-processing technique is modified for estimating the DUT round-trip reflection and transmission functions from the entire signal of system's response, rather than from windowed echos.

Measurement System and Instrument Transfer Functions

The system created by appending a saxophone to the end of a 2-meter cylindrical tube (see Figure 1) is illustrated in Figure 2, where z^{-M} and $\lambda_M(z)$ model the propagation delay and loss in the measurement tube, terminated at one end by the reflection of the speaker $\rho(z)$ and at the other by a 2-port scattering junction, consisting of reflection functions $R_1(z)$ and $R_2(z)$, and transmission functions T_1 and $T_2(z)$, modelling the change of impedance occurring between the cylindrical measurement tube and the predominantly conical saxophone. Following the junction is the propagation delay and loss of the instrument, z^{-N} and $\lambda_N(z)$, respectively, terminated by the bell reflection and transmission functions $R_B(z)$ and $T_B(z)$, which also lump the effects of any open tones holes.

As shown in Appendix A of [2], the transfer function of the entire measurement system with saxophone appended (corresponding to the signal flow diagram in Figure 2), the ratio of the

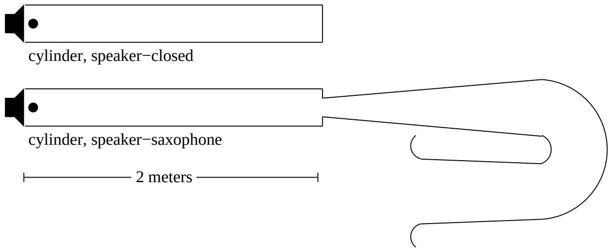

FIGURE 1: The measurement system consisting of a 2-meter tube with a speaker and co-located microphone at one end. The tube is measured first closed (top), then with a saxophone appended (bottom) to produce the measurement's impulse response under both terminating conditions.

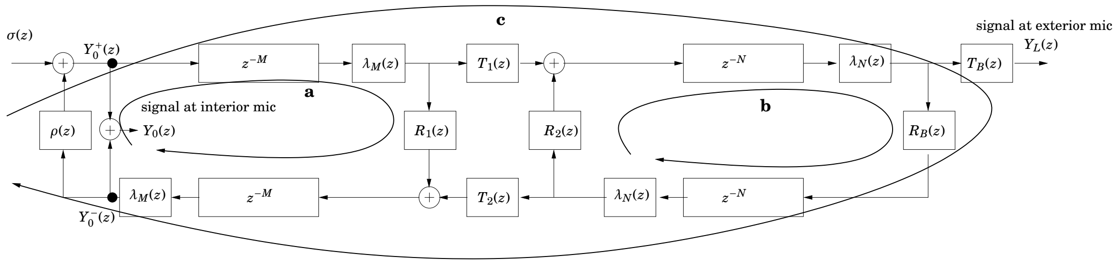

FIGURE 2: The system created by appending a saxophone to a long measurement tube. In addition to the pure delays, functions λ model propagation loss, ρ is the reflection off the speaker, $R_{1,2}$ and $T_{1,2}$ the 2-port scattering junction occurring between a cylinder and predominantly conical section of the saxophone, and R_B and T_B which model the reflection and transmission effects of the bell and open tone holes.

measured signal $Y_0(z)$ to the input speaker transfer function $\sigma(z)$, is given by

$$\begin{aligned} H_0(z) &= \frac{Y_0(z)}{\sigma(z)} \\ &= \frac{1+b_M z^{-2M} + b_N z^{-2N} + b_{M,N} z^{-2(M+N)}}{1+a_M z^{-2M} + a_N z^{-2N} + a_{M,N} z^{-2(M+N)}}, \end{aligned} \tag{5}$$

where the feedforward coefficients are given by

$$\begin{aligned} b_M &= R_1(z)\lambda_M^2(z), \\ b_N &= -R_2(z)R_B(z)\lambda_N^2(z), \\ b_{M,N} &= -[R_1(z)R_2(z) - T_1(z)T_2(z)]R_B(z)\lambda_M^2(z)\lambda_N^2(z), \end{aligned}$$

and the feedback coefficients are given by

$$\begin{aligned} a_M &= -\rho(z)R_1(z)\lambda_M^2(z), \\ a_N &= -R_2(z)R_B(z)\lambda_N^2(z), \\ a_{M,N} &= \rho(R_1(z)R_2(z) - T_1(z)T_2(z))R_B(z)\lambda_M^2(z)\lambda_N^2(z). \end{aligned}$$

Similarly, the transfer function of the measurement system tapped at the position of the microphone outside the bell of the appended saxophone is given by

$$\begin{aligned} H_L(z) &= \frac{Y_L(z)}{\sigma(z)} \\ &= \frac{T_1(z)T_B(z)\lambda_M(z)\lambda_N(z)z^{-(M+N)}}{1+a_M z^{-2M} + a_N z^{-2N} + a_{M,N} z^{-2(M+N)}}, \end{aligned} \tag{6}$$

where the feedback coefficients are as defined in (6).

Conveniently, (5) and (6) can also be expressed in terms of the round-trip reflection of the closed measurement tube,

$$R_{\text{cl}}(z) = \lambda_M^2(z) z^{-2M}, \tag{7}$$

and the round-trip reflection and one-way transmission function of the appended instrument,

$$R_I(z) = R_B(z)\lambda_N^2(z) z^{-2N}, \tag{8}$$

and

$$T_I(z) = T_B(z)\lambda_N z^{-N}, \tag{9}$$

respectively, yielding

$$H_0(z) = \frac{1 + R_1(z)R_{cl}(z) - R_2(z)R_I(z) - (R_1(z)R_2(z) - T_1(z)T_2(z))R_{cl}(z)R_I(z)}{1 - \rho(z)R_1(z)R_{cl}(z) - R_2(z)R_I(z) + \rho(z)(R_1(z)R_2(z) - T_1(z)T_2(z))R_{cl}(z)R_I(z)}, \tag{10}$$

and

$$H_L(z) = \frac{T_1(z)\lambda_M(z) z^{-M} T_I(z)}{1 - \rho(z)R_1(z)R_{cl}(z) - R_2(z)R_I(z) + \rho(z)(R_1(z)R_2(z) - T_1(z)T_2(z))R_{cl}(z)R_I(z)}. \tag{11}$$

This suggests that R_I and T_I can, in turn, be estimated from measurements of R_{cl}, H_0, and H_L, and used to construct the instrument transfer function H given in (2), as well as it's inverse G given in (4), using

$$H(z) = \frac{T_I(z)}{1 - R_M(z)R_I(z)}, \tag{12}$$

and

$$G(z) = \frac{1 - R_M(z)R_I(z)}{T_I(z)}, \tag{13}$$

where $R_M(z)$ is the reflection off the mouthpiece.

Measurement Technique and Post Processing

The system is first measured without a DUT, the end of the 2-M tube opposite the speaker being closed to ensure a perfect reflection. The model of this system can be seen as rigidly terminating the diagram in Figure 2 before the scattering junction, yielding the simplified transfer function

$$H_{0,\text{cl}}(z) = \frac{1 + \lambda_M^2(z) z^{-2M}}{1 - \rho(z)\lambda_M^2(z) z^{-2M}}. \tag{14}$$

The measured response is a sequence of arrivals sufficiently spaced that each may be windowed, and their transforms L_n combined algebraically to yield estimates of the speaker transfer function

$$\hat{\sigma}(\omega) = L_1, \tag{15}$$

and the speaker reflection function

$$\hat{\rho}(\omega) = \frac{\zeta(\omega)}{1 - \zeta(\omega)}, \quad \text{where } \zeta = \frac{L_1 L_3}{(L_2)^2}, \tag{16}$$

where the use of ω is used to indicate measured data, distinguishing it from theoretical expressions given as a function of z. With estimates of measurement system elements (15) and (16), the frequency response of the closed measurement tube may be estimated by

$$\hat{H}_{0,\text{cl}}(\omega) = \frac{L(\omega)}{\hat{\sigma}(\omega)}, \tag{17}$$

where $L(\omega)$ is the transform of entire measurement response of the closed tube, yielding the estimate of the closed-tube round-trip reflection function

$$\hat{R}_{\mathrm{cl}}(\omega) = \frac{\hat{H}_{0,cl}(\omega) - 1}{1 + \hat{\rho}(\omega)\hat{H}_{0,cl}(\omega)}, \tag{18}$$

effectively providing estimates for the combined propagation delay and losses in the measurement tube as described in (7).

A second measurement is then taken, with the DUT appended, yielding a signal at both microphones, $Y_0(\omega)$ and $Y_L(\omega)$. As in (17), the frequency response of the measurement system may be estimated by dividing by the speaker transfer function $\hat{\sigma}(\omega)$ (equivalent to deconvolving in the time domain) to produce $\hat{H}_0(\omega)$—an estimate of $H_0(z)$, and $\hat{H}_L(\omega)$—an estimate of $H_L(z)$. With estimates of the round-trip closed tube reflection function given by (18), it follows from (10) and (11) that the instrument round-trip reflection and one-way transmission are estimated by

$$\hat{R}_I(\omega) = \frac{1 + \dfrac{1 + \hat{\rho}(\omega)\hat{H}_0(\omega)}{1 - \hat{H}_0(\omega)} R_1(\omega)\hat{R}_{cl}(\omega)}{R_2 + \dfrac{1 + \hat{\rho}(\omega)\hat{H}_0(\omega)}{1 - \hat{H}_0(\omega} [R_1(\omega)R_2(\omega) - T_1(\omega)T_2(\omega)]\hat{R}_{cl}(\omega)}, \tag{19}$$

and

$$\hat{T}_I(\omega) = \frac{\hat{H}_L(\omega)\left[1 - \hat{\rho}(\omega)R_1(\omega)\hat{R}_{\mathrm{cl}}(\omega) - R_2(\omega)R_I(\omega) + \hat{\rho}(\omega)(R_1(\omega)R_2(\omega) - T_1(\omega)T_2(\omega))\hat{R}_{\mathrm{cl}}(\omega)R_I(\omega)\right]}{T_1(\omega)\sqrt{\hat{R}_{\mathrm{cl}}(\omega)}}, \tag{20}$$

where $R_{1,2}(\omega)$ and $T_{1,2}(\omega)$ may be obtained theoretically using system dimensions. Equation (19) and (20) may then be used to complete H and G, as given in (12) and (13), respectively.

CONCLUSION AND FUTURE WORK

This process described herein is repeated for each of the possible playable tone hole configurations on the B-flat tenor saxophone, with each fingering applied while the instrument is appended to the measurement tube, and a measurement taken. When estimating the pulse sequence X from the saxophone signal Y_B, a decision must be made regarding which of the several inverse filters G_n should be applied. This is the subject of current work, involving pitch detection, estimation of overblown notes, and calibration of model output to recorded signal.

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REFERENCES

[1] T. Smyth and J. S. Abel, “Toward an estimation of the clarinet reed pulse from instrument performance”, Journal of the Acoustical Society of America **131**, 4799–4810 (2012).

[2] T. Smyth and S. Cherla, “Saxophone by model and measurement”, in *Proceedings of the 9th Sound and Music Computing Conference*, 6 pages (Copenhagen, Denmark) (2012).

[3] T. Smyth and J. Abel, “Estimating waveguide model elements from acoustic tube measurements”, Acta Acustica united with Acustica **95**, 1093–1103 (2009).