A GENERALIZED PARAMETRIC REED MODEL FOR VIRTUAL MUSICAL INSTRUMENTS

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ABSTRACT

A reed, or more generally, a pressure-controlled valve, is the primary resonator for many wind instruments and vocal systems. In physical modeling synthesis, the method used for simulating the reed typically depends on whether an additional upstream or downstream pressure causes the corresponding side of the valve to open or close further.

In this work, a generalized and configurable model of a pressure controlled valve is presented, allowing the user to design a reed simply by setting the model parameters. The parameters are continuously variable, and may be configured to produce blown closed models (like woodwinds or reed-pipes), blown open models (as in simple lip-reeds, the human larynx, harmonicas and harmoniums) and symmetric "swinging door" models. This generalized virtual reed affords the musician the ability to produce a wide variety of sounds which would otherwise only be obtained with several reed instruments.

1. INTRODUCTION

There are several examples of musical instruments (e.g. woodwind and brass) and vocal systems (e.g. the human vocal tract and the avian syrinx) where air pressure from the lungs, or other source, controls the oscillation of a valve by changing the pressure across the valve's reed or membrane to create a constriction through which air flows. Sound sources of this kind are referred to as pressure-controlled valves and they have been simulated in various ways to synthesize virtual musical instruments.

The similarities and differences among various valve geometries lend themselves quite nicely to a single generalized parametric model—one that is completely configurable as determined by the needs of the musician. The generalized model of the valve presented here, and the acoustic tube to which it is connected, is implemented using numerical methods and waveguide synthesis, and runs in the real-time programming environment Pd [1]. We begin by describing the three classes of valves and then discuss how the valve dynamics are generalized to produce a single parametric model. Finally, the musical effects produced by modifying different parameters and changing valve configurations are examined.

2. THE PRESSURE-CONTROLLED VALVE

Pressure-controlled valves are classified according to the effect of an additional pressure applied to the upstream or downstream side of the valve [3, 4]. Fletcher uses the couplet (σ_1, σ_2) to describe the valve behaviour, with $\sigma_i = +1$ signifying an opening of the valve, and $\sigma_i = -1$ signifying a closing of the valve, in response to an upstream ($i = 1$) or downstream ($i = 2$) pressure increase.

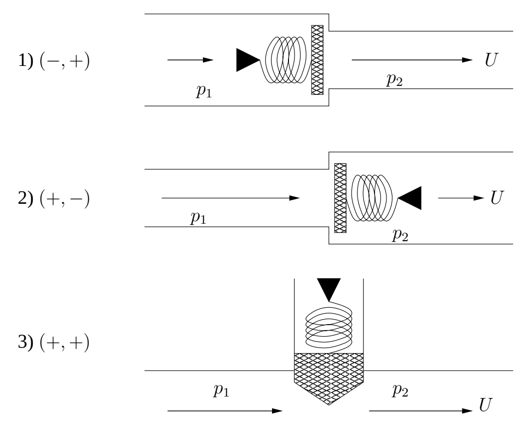

Figure 1. Simplified models of common configurations of the pressure-controlled valve as seen in [4]. 1) $(-,+)$ defines a valve that is blown closed, and is typical of woodwind instruments. 2) $(+,-)$ defines a valve that is blown open, and is exemplified by brass and other lip-reed instruments as well as the human larynx. 3) $(+,+)$ is the principle configuration of the avian syrinx, where an overpressure applied to either side of the valve will cause it to open.

This construction is very useful when evaluating the force driving a mode of the vibrating valve. Consider Fletcher's generalized double reed in a blown open configuration, as shown in Figure 2. In this case, surface S_1 sees an input or upstream pressure p_1, surface S_2 sees the downstream pressure p_2 after flow separation, and surface S_3 sees the flow at the interior of the valve channel and the resulting Bernoulli pressure. With these areas and the corresponding geometric couplet defined, the motion of the

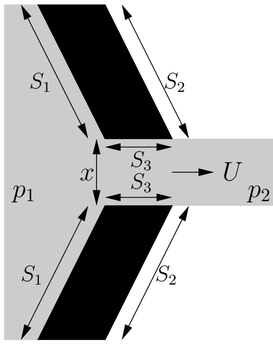

Figure 2. Geometry of a *blown-open* pressure-controlled valve showing effective areas S_1, S_2, S_3 [3].

valve opening $x(t)$ is governed by

$$m\frac{d^2x}{dt^2}+2m\gamma\frac{dx}{dt}+k(x-x_0) = \sigma_1 p_1(S_1+S_3)+\sigma_2 p_2 S_2, \tag{1}$$

where γ is the damping coefficient, x_0 the equilibrium position of the valve opening in the absence of flow, k the valve stiffness, and m the reed mass [3, 4]. The motion equation (1) intentionally does not take into account the force applied by flow for the purpose of simplification.

3. THE GENERALIZED PARAMETRIC MODEL

The generalized parametric model of a pressure controlled valve described below can be configured to operate in any number of ways, allowing the musician the benefit of producing a range of musical effects.

We see from (1) that the behavior of the valve is governed by two features: its dynamics (i.e., how it responds to applied forces), and the manner in which upstream and downstream pressures exert force on the valve. As we will see in §3.2, flow through the valve depends on the valve opening area as a function of time. To develop a generalized pressure controlled valve, therefore, it is desirable to independently control the valve dynamics, the effect of upstream and downstream forces, and the valve area as a function of the valve state.

3.1. Valve dynamics

Figure 3 illustrates one mode of oscillation for each of three possible generalized valve configurations. The displacement of the valve is given by its angle θ from the vertical axis. The configuration of the valve is determined in part by the initial position of the valve θ_0 (its equilibrium position in the absence of flow), and in part by the use of a *stop*—a numerical limit placed at the center vertical axis which prevents the valve from swinging beyond the point $\theta = 0$ and into the shaded region of Figure 3 b) and c).

If no stop is placed, as shown in Figure 3 a) the valve is free to swing across this center boundary and the model provides a symmetric $(+,+)$ type of model, that is, an additional pressure from either side of the valve will cause

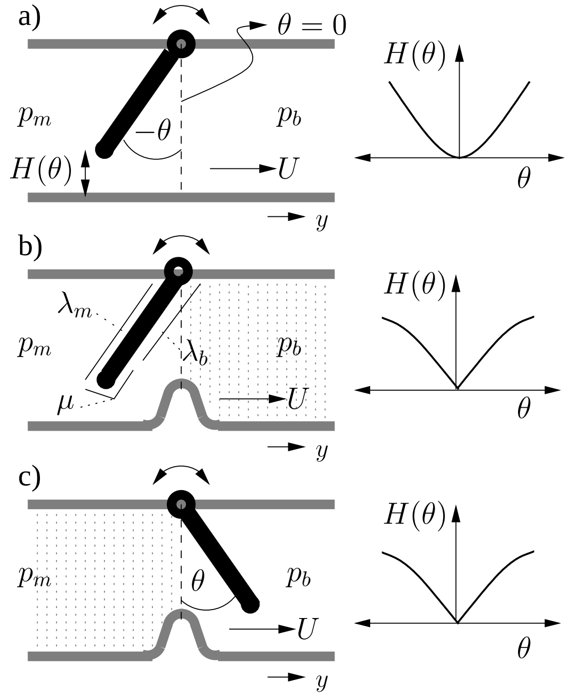

Figure 3. Configurations of the generalized parametric valve, where $H(\theta)$ is a function which determines the height of the valve channel. In configuration a), no stop is specified and the valve swings freely producing a $(+,+)$ symmetric "swinging-door" model. In configurations b) and c), a stop prevents the valve from swinging beyond $\theta = 0$ and into the shaded regions, creating a $(-,+)$ blown closed model and $(+,-)$ blown open model respectively.

it to open further. If a stop is placed in the channel, the configuration is further determined by the initial equilibrium position of the valve θ_0. If the valve's initial position is to the left of the center ($\theta_0 < 0$), a pressure increase from the air source will cause the valve to close further and a pressure increase from the bore will cause it to open further. This creates a $(-,+)$ *blown closed* model similar to woodwind instruments and the valve shown in Figure 1, b). Contrarily, if the valve's initial position is to the right of the stop point ($\theta_0 > 0$), a pressure increase from the air source will cause the valve to open further and a pressure increase from the bore will cause it to close further. This creates a $(+,-)$ *blown open* model similar to lip-reeds and shown in Figure 1, c).

Once the valve is set into motion, the value for θ is determined, for small displacements, by the familiar second order differential equation

$$m\frac{d^2\theta}{dt^2} + m2\gamma\frac{d\theta}{dt} + k(\theta-\theta_0) = F, \tag{2}$$

where k is the stiffness of the reed, γ and m are defined as above, and F is the overall driving force acting on the reed. The fundamental frequency of valve vibration (resonance frequency) is given by $\omega_v = \sqrt{k/m}$. In the generalized model, the displacement of the valve is determined by first considering the force F in (2).

Let us assume the valve reed is hinged as in Figure 3. Let λ_m be the effective length of the valve which sees the mouth pressure p_m, λ_b be the length of the valve which sees the bore pressure p_b, and μ the length of the valve that sees the flow. There is a force in the positive θ direction, on the surface area $\lambda_m w$, given by

$$F_m = w\lambda_m p_m, \tag{3}$$

where w is the width of the valve channel. There is also a force on the bore side of the reed away from the jet, given by

$$F_b = -w\lambda_b p_b, \tag{4}$$

forcing the valve in the negative θ direction when $p_b > 0$. The force applied by the flow (which also forces the reed open) is found by integrating the pressure along the flow and is given by

$$F_U = \text{sign}(\theta) w\mu \left(p_m - \frac{\rho}{2}\left(\frac{U}{A}\right)^2 \right), \tag{5}$$

where A is the cross-sectional area of the valve channel given by wH, H is the opening of the valve (see Figure 3) calculated from θ and the geometry of the valve, and sign(θ) determines the direction of the force: if $\theta > 0$, the force acts in the positive θ direction and if $\theta < 0$ the force acts in the negative θ direction. The force F acting on the valve is then obtained by summing (3), (4) and (5) and is given by

$$F = w\lambda_m p_m + \text{sign}(\theta) w\lambda_b p_b - w\mu \left(p_m - \frac{\rho}{2}\left(\frac{U}{A}\right)^2 \right). \tag{6}$$

To specify the valve classification, the musician need only specify the equilibrium position θ_0 and whether the valve should be limited by $\theta = 0$ (for the blown open and closed cases). It may also be desirable to create a stop point that is valid only under certain conditions: overblowing, for example, could cause the valve to beat against the stop with enough force to push it past the limit, effectively blowing the valve into a new configuration. This and other variations could be implemented by making the reed stiffness k a function of valve angle θ.

3.2. Volume flow

Many valve models (and particularly clarinet reeds) are implemented using a lookup table which matches the value for flow with the pressure drop across the valve [2, 4]. This is known as the quasi-static, Bernoulli-flow model because the value of flow U is established by relating the pressure difference and the volume flow under constant-flow conditions. Though this implementation has produced satisfactory sound at low computational cost, it is not suitable for a generalized model as it is not physically accurate and does not provide access to certain desired parameter values.

The dynamic model, which replaces the static table with a differential equation for volume flow, was presented in [5, 6] and permitted the development of the *feathered* reed—a smoothing of the volume flow cutoff between open and closed valve states. Within the context of the generalized valve, the dynamic model now also has the added benefit of allowing for valve modifications in real-time.

The flow is in contact with the surface of the reed for a distance μ, beyond which it is assumed that the flow separates and forms a jet. For this reason, we are interested in the differential flow *before* that point. The force on a thin slice dy along the valve channel is given by

$$F = A(y;\theta)\Delta p(y). \tag{7}$$

where $A(y;\theta)$ is the cross section area of the valve channel at a point y along the channel for the generalized reed at an angle θ, and where $\Delta p(y)$ is the pressure drop across this section of the reed. The force is applied to a volume of air $A(y;\theta)dy$ having mass

$$\rho A(y;\theta)dy, \tag{8}$$

where ρ is the air density. Newton's second law—force is the product of mass and acceleration—can then be applied to (7) and (8) to obtain

$$A(y;\theta)\Delta p(y) = \rho A(y;\theta)dy\frac{dv}{dt}, \tag{9}$$

where acceleration is given by the time derivative of the particle velocity, dv/dt, and is assumed constant over this section dy of the reed. We can then substitute particle velocity for volume flow scaled by area and integrate over the length of the channel to obtain

$$p(0) - p(\mu) = \rho\frac{dU}{dt}\int_{y=0}^{y=\mu} dy/A(y;\theta), \tag{10}$$

where $y = 0$ is the channel entrance and $y = \mu$ is the point of flow separation. Bernoulli's equation is used to calculate the pressure entering the valve, p_0, and then the pressure at flow separation $p(\mu)$ is replaced with the bore pressure p_b to obtain the differential equation governing volume flow

$$\frac{dU}{dt} = (p_m - p_b)\frac{A(x)}{\mu\rho} - \frac{U^2}{2\mu A(\theta)}, \tag{11}$$

where the flow is assumed to be in contact with the reed for a distance μ.

The singularity in (11) as the valve opening approaches zero makes clear the need for a *feathered* valve, a method fully developed for the avian syrinx [5] and the clarinet reed [6] using the small area solution for $dU(t)/dt$. The update governing air flow is given by

$$U(t+1) = U(t) + (p_m - p_b)\frac{A(t)T}{\mu\rho} - \frac{U(t)^2 T}{2\mu A(t) + U(t)T}, \tag{12}$$

where $A(t)$ is the valve channel area at time t, and T is the sampling interval.

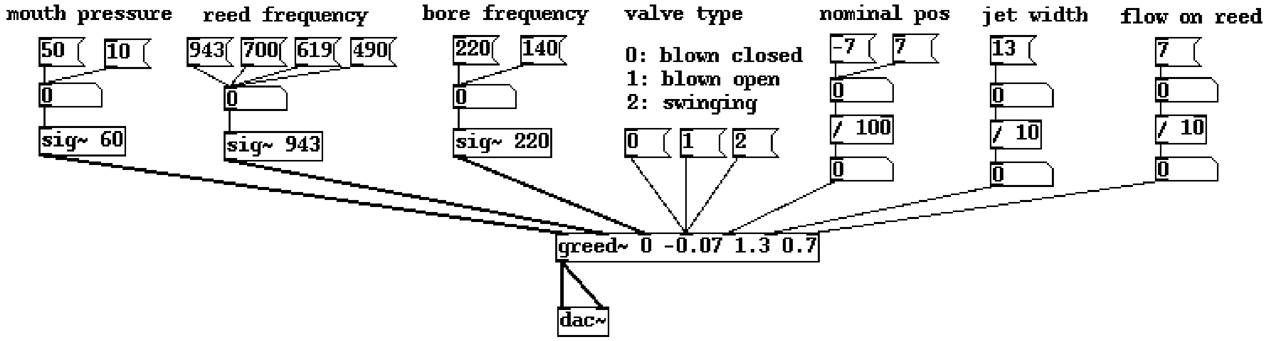

Figure 4. Pd object showing input parameters.

It is clear that the valve channel area is critical to the volume flow and the sound of the instrument. As the reed angle θ changes, the valve opening area changes according to the changing valve channel height, $A(\theta) = wH(\theta)$. As illustrated in Figure 3, any number of channel area functions are possible by choice of channel profile, and, in particular, by the channel height function $H(\theta)$. Setting $H(\theta) = |\sin\theta|$ for example, approximates the channel area of a clarinet reed, whereas the function $H(\theta) = 1 - \cos\theta$ approximates that of a lip reed.

4. EXAMPLE AND CONCLUSIONS

The model was implemented in Pd [1] and takes arguments as shown in Figure 4. Figure 5 shows spectrograms of the sound produced in response to a burst of mouth pressure for blown-closed (top) and blown-open (bottom) valves attached to identical bores. In both cases, the reed and resonant frequencies, f_r and f_b respectively, were quite different with $f_r \gg f_b$ as is typically the case. As expected, in the blown closed case the reed frequency f_r has very little effect on the sounding frequency which, as seen by the spectrogram, is closer to the frequency of the bore f_b. Also as anticipated, in the blown open case the sounding frequency is closer to the reed resonant frequency f_r.

The generalized pressure controlled valve presented here is capable of expressing any of the three valve classes—blown open, blown closed, and a symmetric model—by changing the model parameters. The classes have significantly different pitch and timbre characteristics, made even more so by changing certain of the model's geometric quantities. This model contributes a very useful tool for creating, and blending among, a variety of instrument sounds under real time control.

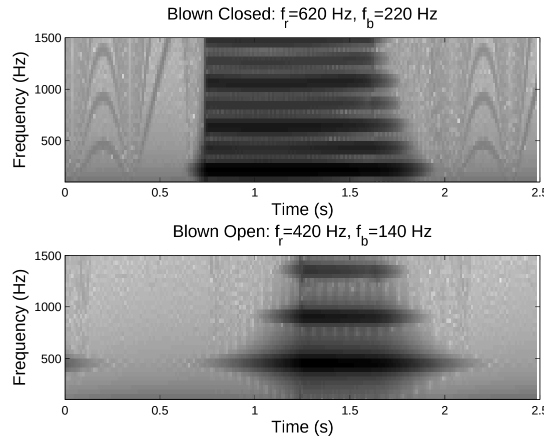

Figure 5. In the blown closed case, top, the sounding frequency, as seen by the spectrogram, is closer to the frequency of the bore f_b. In the blown open case, bottom, the sounding frequency is closer to the frequency of the reed f_r.

5. REFERENCES

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