EXTENDING THE GENERALIZED REED MODEL WITH MEASURED REFLECTION FUNCTIONS

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ABSTRACT

In previous work, the authors presented a generalized parametric model of a pressure controlled valve, allowing the user to design a continuum of reed configurations, including "blown open", "blown closed" and the "swinging door". Though the generalized reed model behaved as expected, the quality of the produced sound was somewhat limited, likely due to the dependence of reed oscillation on the connected instrument bore and bell.

In this work we further explore the sound production of the generalized reed by incorporating reflection filters measured from actual musical instruments. The measurement technique is shown to produce results closely matching theoretical expectation for cylindrical and conical tubes, and is applied to the clarinet and trumpet. Measurements are incorporated into a waveguide model using the generalized reed.

1. INTRODUCTION

A physics-based synthesis of a reed instrument typically involves simulating the dynamics of a pressure-controlled valve, and coupling the result to a model of the propagating pressure waves travelling along the instrument bore. In many cases the bore is modelled using digital waveguide synthesis [1], that is, the right and left travelling pressure waves are modelled using a bi-directional delay line and filter elements accounting for losses occurring during propagation and at boundaries.

In previous work, the authors presented a model for the reed element, and in particular a generalized parametric model of a pressure controlled valve, allowing the user to design a continuum of reed configurations, including "blown open", "blown closed" and the "swinging door" (reviewed in Section 2). Though the generalized reed model behaved as expected, the quality of the produced sound was somewhat limited, likely due to the simplicity of the connecting tube employed. Since the nature of the tube and bell, and the resulting pressure fluctuations at the mouthiece, influence the opening and closing of the reed, an improved synthesis is expected by using more accurate bore and bell models.

The oscillation of a "blown open" reed, such as one might find in a trumpet or other lip-reed instrument, is strongly coupled to the bore, making playability highly dependent on the bore resonances. Any trumpet player will agree that initiating, and sustaining, oscillations of the lips when blowing into a cylindrical tube is considerably different than when blowing into the bore of a trumpet, with its flared opening serving to "shift" resonant peaks. Coupling a simulation of a "blown open" reed to a model of a cylindrical bore would present similar difficulties, and in particular, would not yield the quality of sound one might expect from a trumpet, even if oscillation were achieved. A parametric change in the configuration of the generalized reed model would, therefore, also likely require a change in the model of the bore.

In an approach similar to that described in [2], in this work we further explore the sound production of the generalized reed by incorporating measured reflection filters corresponding to different musical instrument bores. The measurement technique was originally demonstrated by the authors using simple cylindrical and conical tubes (and their combination), and and produced results closely matching theory [3].

2. THE GENERALIZED REED MODEL

In reed instruments, as well as many vocal systems, air pressure from a source such as the lungs controls the oscillation of a valve by creating a difference between its upstream (incoming) and downstream (outgoing) pressure. This primary resonator, known as a pressure-controlled valve, is classified according to its behaviour in the presence of additional upstream or downstream pressure [4]. If an increase in blowing pressure causes the valve to close further, and a bore pressure increase causes the valve to open further, the reed is said to be *blown closed*, the classification of most woodwind instruments. If a blowing pressure increase causes the valve to open further, and an increase in bore pressure causes the valve to close, the reed is *blown open*, the typical configuration of brass (lip reed) instruments, and the human voice. A *swinging door* or "transverse" reed, typically found in the avian syrinx, is one where a pressure increase from either side of the valve will cause it to open further.

The generalized reed model was first introduced in [5], providing a configurable model of a pressure controlled valve, allowing the user to design their own virtual reed, simply by setting model parameters. The parameters are continuously variable, and may be configured to produce any of the three aforementioned valve classifications, as well as setting the valve geometry. Figure 1 illustrates one mode of oscillation for each of the three possible classifications, with the displacement of the valve being given by its angle θ from the vertical axis.

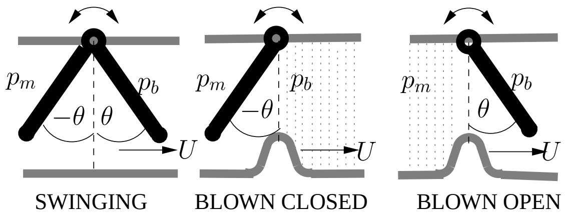

Figure 1. The valve types, showing four evolving model parameters: input mouth pressure p_m, bore pressure p_b, volume flow U, and valve displacement $theta$ (which is constrained differently for each type).

The valve classification is determined in part by its initial position θ_0 (its equilibrium position in the absence of flow), and in part by the use of a *stop*—a numerical limit placed at the center vertical axis that prevents the valve from swinging beyond the point $\theta = 0$ (see Figure 1 b and c). If no stop is placed, as shown in Figure 1 a), the valve is free to swing across this center boundary and the model provides a symmetric "swinging" model, that is, an additional pressure from either side of the valve will cause it to open further. If a stop is placed in the channel, the configuration is further determined by the initial equilibrium position of the valve θ_0: an initial position to the left of the stop, at $\theta_0 < 0$, will cause the reed to *blow closed*, while an initial position to the right of the stop, $\theta_0 > 0$, will cause the reed to *blow open*. A clarinet classification, for example, is implemented with $\theta_0 < 0$ plus a stop.

The geometry of the valve may be further specified by setting the effective length of the reed that sees the mouth pressure λ_m, the reed length that sees the bore pressure λ_b, and the reed length that sees the flow, given by μ. These variables have an audible effect on the overall driving force acting on the reed, given by F in (1), and can be seen as offering finer control of embouchure.

Once the valve is set into motion, the value for θ is determined by the second order differential equation

$$m\frac{d^2\theta}{dt^2} + m2\gamma\frac{d\theta}{dt} + k(\theta - \theta_0) = F, \qquad (1)$$

where m is the effective mass of the reed, γ is the damping coefficient, k is the stiffness of the reed, and F is the overall driving force acting on the reed, a function of the mouth and bore pressure, and flow in contact with the reed. The frequency of vibration for this mode is given by $\omega_v = \sqrt{k/m}$.

Discretization using the trapezoidal rule for numerical integration, yields the transfer function

$$\frac{X(z)}{F(z) + kx_0} = \frac{1 + 2z^{-1} + z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}, \qquad (2)$$

and the corresponding difference equation

$$\begin{aligned} x(n) &= [F_k(n) + 2F_k(n-1) + F_k(n-2) - \\ &\quad a_1 x(n-1) - a_2 x(n-2)]/a_0, \end{aligned} \qquad (3)$$

where $F_k(n) = F(n) - kx_0$, and

$$\begin{aligned} a_0 &= mc^2 + mgc + k, \\ a_1 &= -2(mc^2 - k), \\ a_2 &= mc^2 - mgc + k. \end{aligned}$$

This discretization is equivalent to applying a bilinear transform. Since pole frequencies are well below the Nyquist limit (half the sampling rate), they is no need for prewarping.

The force driving the reed F is equal to the sum of the forces acting on the reed, $F = F_m + F_b + F_U$, where $F_m = w\lambda_m p_m$ is the force acting (in the positive θ direction) on the surface area $\lambda_m w$, $F_b = -w\lambda_b p_b$, is the force acting (in the negative θ direction) on the surface area $\lambda_b w$, and F_U is the force applied by the flow (which forces the reed open) given by

$$F_U = \text{sign}(\theta) w\mu \left(p_m - \frac{\rho}{2}\left(\frac{U}{A}\right)^2 \right). \qquad (4)$$

As can be seen by (4), the total force driving the reed is dependent on the valve classification, since the sign of θ is determined by its limits.

The differential equation governing air flow through the valve, fully derived in [6], is given by

$$\frac{dU}{dt} = (p_m - p_b)\frac{A(t_0)}{\mu\rho} - \frac{U(t_0)^2}{2\mu A(t_0) + U(t_0)T}. \qquad (5)$$

where p_m is mouth pressure, p_b is the bore pressure (see discussion in the following section), $A(t)$ is the cross sectional area of the valve channel, and μ is the length of reed that sees the flow. Equation (5) is used to update the flow U every sampled period (given by the inverse of the sampling rate).

There are, therefore, three variables that evolve over time in response to an applied pressure p_m: the displacement of the reed θ (determined using 3), the flow U, determined using the update given by (5), and the pressure at the base of the bore p_b, obtained using waveguide synthesis incorporating a low-latency convolution [7] of the measured reflection and/or loss functions.

3. SYNTHESIS OF THE BORE

3.1. Waveguide Synthesis of Instrument Bore

In the original presentation of the generalized reed, the pressure at the base of the bore p_b is determined using waveguide synthesis incorporating theoretical reflection filters at the open end boundary. The model, as seen in Figure 2, consists of a bi-directional delay line to account for the pure delay of the bore (a function of either the bore length or the desired frequency), as well as digital filters to account for losses occurring during propagation, and at boundary reflections.

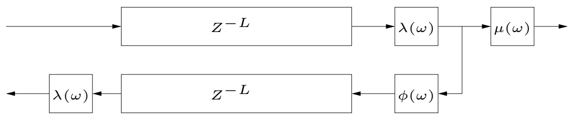

Figure 2. A waveguide model of a cylindrical tube with commuted wall loss filters, $\lambda(\omega)$, at upper and lower delay line observation points, a reflection filter $\phi(\omega)$ and a transmission filter $\mu(\omega)$.

Because oscillation of the reed is dependent on the shape of the bore, and its associated resonances, this waveguide model alone was insufficient for obtaining the various possible sounds of which a generalized reed should be capable. We therefore incorporate the results of acoustic tube measurements to obtain reflection (and corresponding transmission) filters more appropriate to a chosen valve classification.

3.2. Response Measurements of Instrument Bores

In work originally presented at [3], the authors presented a measurement technique for obtaining very accurate measurements of simple acoustic tubes (cylindrical and conical), and which allowed for observation, and estimation, of each of the waveguide elements seen in Figure 2. To isolate each waveguide model element, a system was developed using a series of four tube structures (see Figure 3) and then extended to include both clarinet and trumpet bores (see Figure 4) to obtain their reflection functions.

1. **Cylinder—closed end.** A 2 meter long cylinder is closed at one end to ensure a perfect reflection, allowing for calibration to the speaker output, speaker reflection, and propagation (wall) loss filter transfer functions. This measurement can later be deconvolved from one with an appended instrument to obtain the reflection function for that instrument.

2. **Cylinder—open end.** Opening the cylinder allows for estimation of the transfer function for the reflection at the open end of a cylinder ($\sigma(\omega)$ in Figure 2). If a tube structure (such as an instrument) is appended to the cylinder, $\sigma(\omega)$ represents instead the reflection function of that strucutre.

3. **Cylinder+Cone (Cylicone)—closed end.** The addition of a conical flare with a spherical termination (to ensure a perfect reflection of spherical waves) allows for estimation of reflection and transmission filter transfer functions at the junction.

4. **Cylinder+Cone (Cylicone)—open end.** Opening the conical end allows for estimation of the reflection function the cone.

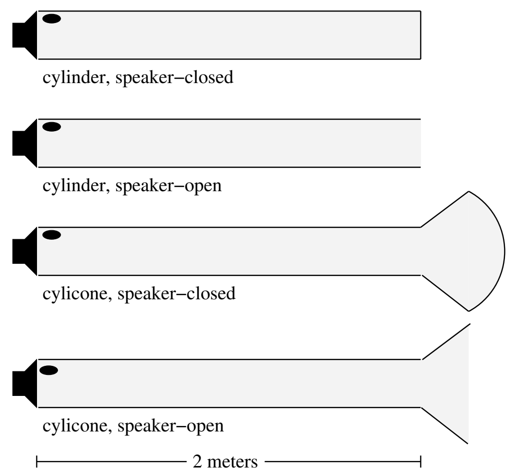

Figure 3. Four tube structures incorporating open and closed boundaries and cylindrical and conical sections.

Results are best presented for the cylinder and cylicone cases, as these structures can be easily modeled using theoretically-based waveguide synthesis techniques, and thus provides a good basis for comparison between model and measurement (Figure 5). The measurements yield data (from which waveguide model elements are estimated) that is shown to be almost identical to the output of the theoretically-based waveguide model (see Figure 5), allowing the technique to be confidently expanded to include the reflection functions of actual instruments.

Figure 4 shows a clarinet and trumpet bore connected to the end of the calibration tube. Appending an instrument is much like appending the cone, and using existing responses of the closed cylindrical tube, will allow for estimation of the response of an impulse travelling the length of the bore, reflecting at its open flared end, and then returning to the junction with the cylinder. The reflection function magnitudes for the clarinet bell and the

Figure 4. The clarinet and trumpet reflection functions are measured by appending them to the calibration cylinder.

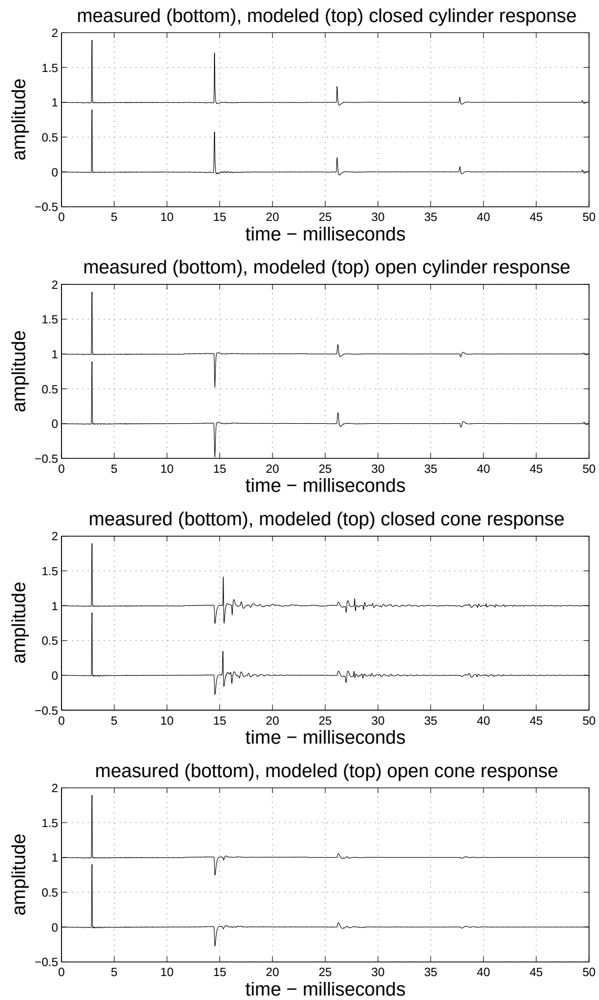

Figure 5. Measured impulse responses (bottom of each figure), and those produced based on theoretical consideration (top of each figure), show excellent agreement.

trumpet mouthpiece are shown in Figure 6, the former having the expected low-pass characteristic. In addition to measuring bore and instrument reflection functions, other characteristics may be measured. For instance, Figure **??** shows a measured trumpet mouthpiece reflection function which may be used to determine the mouthpiece input impedance.

The reflection function is incorporated into the waveguide model by convolving the right traveling wave with the reflection impulse response to produce the left, incoming, pressure wave traveling toward the reed. The required convolution may be computed using frequency-domain techniques, and to ensure the same level of interactive control as the waveguide model, a low-latency implementation [7] may be used.

4. CONCLUSION

We have proposed a generalized reed with considerable potential flexibility, in that it can be configured to model any of three different valve types. In connecting it to a simple generic tube model however, done originally for the purpose of keeping the conditions consistent while comparing the behaviors of different reed classifications, we found that the quality of the produced sound was somewhat limited. By having more appropriate variation in the bore and bell, obtained using measured reflection functions of actual instruments, we can improve upon the coupling with the reed and the overall produced sound.

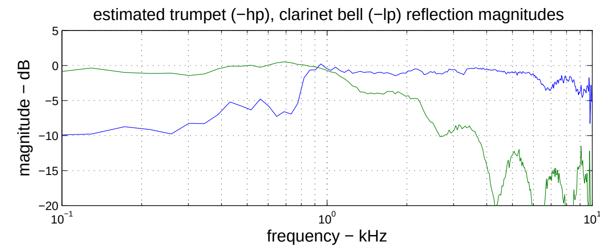

Figure 6. The reflection function magnitudes for the clarinet bell (lp = lowpass) and the trumpet mouthpiece (including mouthpiece) (hp = highpass).

5. REFERENCES

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