Generative Feedback Networks Using Time-Varying Allpass Filters

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ABSTRACT

Audio signal processing applications of first and second-order allpass filters are discussed. Stability issues can arise when second-order allpass filters are made time varying. This is demonstrated analytically and experimentally. A solution, in the form of a power-preserving matrix formulation of the second-order allpass is presented. This filter is studied with applications to feedback systems and generative synthesis.

1. INTRODUCTION

Allpass filters are well-known signal processing tools, commonly employed in audio synthesis and processing for their frequency-dependent phase shift and unity magnitude response. They have been used widely in applications such as reverberation [1], physical modeling [2], spectral delay filters [3], and modulation effects [4, 5, 6].

Problems can arise however, when traditional, time-invariant filter structures are made time-varying. Filters that are stable with a given set of coefficients can quickly experience rapid growth in output power when those coefficients are allowed to vary over time—even if only moving to another set of "stable" coefficients. This presents challenges for their use in certain contexts, and indeed for their application to the feedback networks and *generative audio systems* discussed herein.

In this paper, "generative audio system" is a term used by the authors to specify a particular subset of generative music systems. Any generative music system depends on both sonic materials and the principles by which they are organized. Generative *audio* systems, as coined here, have a tight coupling between these two elements, and are related to Holopainen's "autonomous instruments" [7]. That is, the same process governing high-level musical details—like form, dynamics, and timbre—should also generate the audio signal in which those details are embodied. Such systems are capable of producing dynamic and surprising audio output with little or no human input, and function entirely at the signal rate. The generative process is not based on manipulations of symbolic data, but instead functions directly on the output of audio synthesis or processing algorithms. Allpass filters, particularly in their time-varying form, produce very effective generative behavior when placed in feedback networks and their parameters are modulated in a variety of ways. Modulation of parameters, however, necessitates a change in allpass filter coefficients over time, and thus a handling of the instabilities arising in such cases by use of a time-variant form.

This paper begins by reviewing first and second-order allpass filters and their application to a phase-distortion effect—previous work on which this paper builds by handling the instabilities caused by modulating allpass filter coefficients over time. The presence of instabilities in the time-varying case is illustrated in Section 3, along with an analysis method for predicting stability of *periodically* time-varying coefficients. It is then shown in Section 4 that a power-preserving rotation matrix formulation of the second-order allpass, which can be made-equal to the time-invariant case, ensures stability. Finally, the use of the time-variant filter in a generative audio system is explored in Section 5.

2. SYNTHESIS APPLICATIONS OF ALLPASS FILTERS

2.1 First and Second-order Allpass Filters

Allpass filters are well known for applying a frequency-dependent delay to an input signal, while leaving its magnitude spectrum unmodified. The first-order allpass filter has a difference equation given by

$$y(n) = ax(n) + x(n-1) - ay(n-1), \tag{1}$$

where $|a| < 1$ for stability and for the filter to impart unity gain at all frequencies (the characteristic after which the filter is named). The phase behaviour can be controlled to some extent by specifying a frequency $f_{\pi/2}$ (in Hz) at which $\pi/2$ phase shift is reached (the frequency at which the angle of the filter's frequency response is $-\pi/2$) and setting the allpass filter coefficient to

$$a = \frac{\tan(\pi f_{\pi/2}/f_s) - 1}{\tan(\pi f_{\pi/2}/f_s) + 1}, \tag{2}$$

where f_s is the sampling rate used.

Though (2) offers some ability to specify phase behaviour, additional control is afforded in the case of the second-order allpass filter,

$$\begin{aligned} y(n) \quad = \quad & -cx(n) + d(1-c)x(n-1) + x(n-2) - \\ & d(1-c)y(n-1) + cy(n-2), \end{aligned} \tag{3}$$

where c and d are set according to the desired "bandwidth" f_b of the phase transition region and the frequency f_π at which the phase response is $-\pi$:

$$d = -\cos\left(\frac{2\pi f_\pi}{f_s}\right) \quad \text{and} \quad c = \frac{\tan(\pi f_b/f_s) - 1}{\tan(\pi f_b/f_s) + 1}. \tag{4}$$

Adjusting f_π and f_b allows for both placement of the frequency point at which a phase shift of π is reached and control over the slope of the phase transition region. Figure 1 shows the effects of f_π and f_b on the phase response.

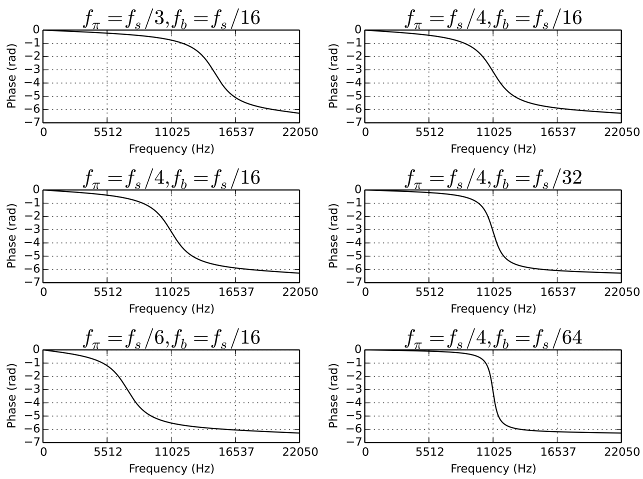

Figure 1. Effects of f_π and f_b on the phase response of the second-order allpass. In the left column, f_π changes while f_b remains constant. In the right, f_b changes while f_π remains constant.

2.2 Frequency-selective Phase Distortion

In [6], a technique was described in which a user may apply a vibrato or phase-distortion effect to particular frequency bands of a complex sound, leaving the rest of the spectrum relatively unmodified. The technique involved sinusoidally modulating the parameter f_π,

$$\tilde{f}_\pi(n) = f_\pi + M\cos\left(\frac{2\pi f_m n}{f_s}\right), \tag{5}$$

where $\tilde{\cdot}$ indicates a function made time varying, M is the depth of modulation, and f_m is the modulation frequency. The result is a second-order allpass similar to (3), but made time varying,

$$\begin{aligned} y(n) \quad = \quad & -cx(n) + \tilde{d}(n)(1-c)x(n-1) + x(n-1) - \\ & \tilde{d}(n)(1-c)y(n-1) + cy(n-2), \end{aligned} \tag{6}$$

where

$$\tilde{d}(n) = -\cos\left(\frac{2\pi \tilde{f}_\pi(n)}{f_s}\right). \tag{7}$$

By applying carefully designed modulation to the center frequency f_π, it is possible to apply a frequency-selective phase distortion effect. Due to the non-linearity of the phase response, certain frequency bands (those nearest to f_π) are modulated while others are left unmodified.

The results in [6] showed very promising musical potential, but use of a "time-invariant" filter caused expected instabilities. The work presented here addresses these issues so that the original synthesis technique may be used more reliably and its musical potential further explored.

3. STABILITY ANALYSIS OF TIME-VARYING ALLPASS FILTERS

3.1 Instability of the Time-Invariant Allpass Filter

It is well-known that systems with constant parameters can become unstable when those parameters are made time varying. In [8], it is shown that if the first-order allpass filter (1) is made time-varying,

$$y(n) = a(n)x(n) + x(n-1) - a(n)y(n-1), \tag{8}$$

the filter coefficient condition $|a(n)| \leq \epsilon < 1$ is not sufficient to ensure energy preservation. It is demonstrated that when $x(n)$ is an impulse and filter coefficients $a(n) = \epsilon(-1)^n$, the output energy is

$$\begin{aligned} ||y(\cdot)||^2 \quad &= \quad a(0)^2 + (1 - a(0)a(1))^2 \left(1 + \sum_{n=2}^{\infty} \prod_{i=2}^{n} a(i)^2\right) \\ &= \quad \epsilon^2 + (1+\epsilon^2)^2 \sum_{n=0}^{\infty} \epsilon^{2n} \\ &= \quad \frac{1+3\epsilon^2}{1-\epsilon^2}, \end{aligned} \tag{9}$$

which is greater than the input for any choice of $\epsilon < 1$ [8], and thus contradicting the all-pass filter assumption that all frequencies pass with equal (unity) gain. A solution is suggested in [8] in terms of a wave digital one port and a corresponding orthogonal matrix formulation that ensures energy preservation—a solution that is *almost* equivalent to the power preserving rotation matrix to be discussed in Section 4.

3.2 Stability Analysis

Though the above example shows that stability cannot be ensured at the Nyquist frequency, it is not the case that all time-varying coefficients will render (6) unstable. The coefficients in (6) are modulated sinusoidally enabling the use of a technique for determining stability of a *periodic* time-varying system by representing the system as a state space matrix [9]. It should be noted that the use of a sinusoidal modulation signal is not a requirement for stability, and the discussion here follows [6] where sinusoidal terms were used to simplify analysis of spectral components added by phase distortion.

It is necessary to represent only the recursive part of (6) as a system of equations which, in matrix form, becomes

$$\begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix} = \begin{bmatrix} -d(n)(1-c) & c \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} y_1(n-1) \\ y_2(n-1) \end{bmatrix}, \tag{10}$$

or

$$\mathbf{y}(n) = \mathbf{A}(n)\mathbf{y}(n-1), \tag{11}$$

where $\mathbf{y}(n)$ and $\mathbf{y}(n-1)$ are state vectors of the system at time sample n and $n-1$, respectively, and $\mathbf{A}(n)$ is the time-varying coefficient matrix. The filter state vector at time sample k may be determined by

$$\begin{aligned} \mathbf{y}(k) &= \mathbf{A}(k)\mathbf{y}(k-1) \\ &= \mathbf{A}(k)\mathbf{A}(k-1)\mathbf{y}(k-2) \\ &= \ldots \\ &= \prod_{n=k}^{1} \mathbf{A}(n)\mathbf{y}(0), \end{aligned} \tag{12}$$

for known initial conditions $\mathbf{y}(0)$. Though stability can be calculated by assessing the behavior of the state vector's norm,

$$||\mathbf{y}(k)||^2 = y_1^2(k) + y_2^2(k) + \ldots + y_n^2(k), \tag{13}$$

a simplified method can be used if filter coefficients change periodically. For a periodically linear time-variant system, the system monodromy matrix,

$$\mathbf{C}(N,0) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \prod_{i=N}^{1} \mathbf{A}(n). \tag{14}$$

connects arbitrary states of the system separated by N, the period of variation, in samples, of $d(n)$. The system is stable if the absolute values of all eigenvalues $|\lambda_1, \lambda_2, \ldots, \lambda_n|$ of $\mathbf{C}(N,0)$ are less than or equal to 1, giving the following limits which ensure stability:

$$\begin{aligned} 1 - C_{11} - C_{22} + \det(\mathbf{C}(N,0)) &\geq 0 \\ 1 + C_{11} + C_{22} + \det(\mathbf{C}(N,0)) &\geq 0 \\ |\det(\mathbf{C}(N,0))| &\leq 1, \end{aligned} \tag{15}$$

where

$$\det[C(N,0)] = \prod_{i=N}^{1} c = c^N. \tag{16}$$

By evaluating the conditions in (15), it is possible to determine whether a given set of filter parameters f_m, f_π, M, f_b, will produce a stable, periodic time-varying filter. Figure 2 shows the complexity of the interactions between the four parameters. The interdependence of these parameters makes it difficult to intuitively understand which combinations will produce instability, rendering this version of the filter very dangerous for real-time use where a variety of settings may be used.

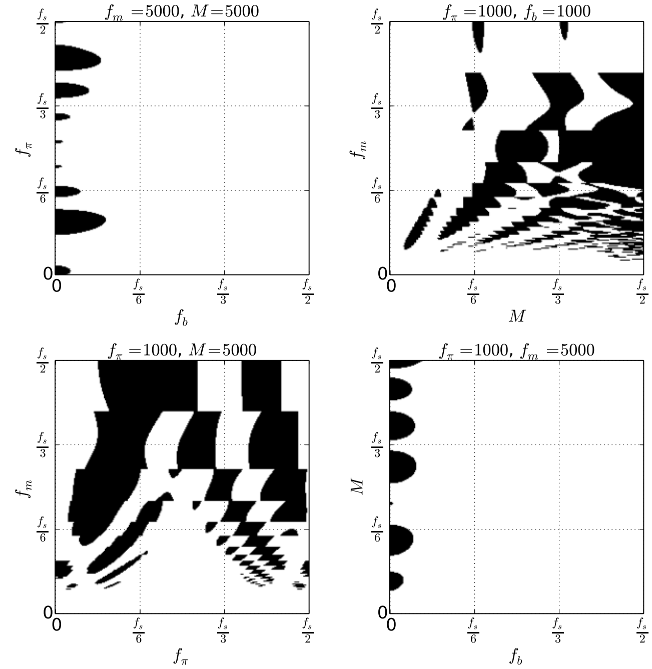

Figure 2. Influence on stability of the interaction between pairs of parameters. Each subplot allows two parameters to vary, while the others are fixed. White indicates a stable combination.

4. A POWER-PRESERVING ALLPASS FILTER

Though it is possible to analyze stability and determine which combinations of parameters f_π, f_b, M, and f_m will produce a system in which power is preserved, there is an alternate power-preserving form of the first and second-order allpass filters. Fortunately, for the application considered here, the power-preserving allpass filter exhibits comparable phase response characteristics, and can be parameterized in nearly the same way as the time-invariant case.

Ensuring the output power of a filter is equal to the input power,

$$\sum_{n=0}^{N-1} x^2(n) = \sum_{n=0}^{N-1} y^2(n), \tag{17}$$

can be accomplished by using a standard power-preserving rotation matrix such as the one used in [10]:

$$\begin{bmatrix} \cos(r) & -\sin(r) \\ \sin(r) & \cos(r) \end{bmatrix}. \tag{18}$$

If multiple input signals to a system $x_1(n), \ldots, x_K(n)$ are considered coordinates in K-space, then a rotation is an operation that, by definition, will preserve the length, or correspondingly, the magnitude of the input [11].

The second-order allpass filter is obtained using a pair of power-preserving rotation matrices, yielding

$$\begin{bmatrix} y(n) \\ z_1(n) \\ z_2(n) \end{bmatrix} = \begin{bmatrix} x(n) \\ z_1(n-1) \\ z_2(n-1) \end{bmatrix} \mathbf{DE}, \tag{19}$$

where

$$\mathbf{D} = \begin{bmatrix} \cos(r_1) & -\sin(r_1) & 0 \\ \sin(r_1) & \cos(r_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{20}$$

and

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(r_2) & -\sin(r_2) \\ 0 & \sin(r_2) & \cos(r_2) \end{bmatrix}. \tag{21}$$

With some algebraic manipulation, the equivalent difference equation can be obtained

$$\begin{aligned} y(n) &= b_0 x(n) - b_1 x(n-1) + x(n-2) + \\ &\quad b_1 y(n-1) - b_0 y(n-2), \end{aligned} \tag{22}$$

where

$$\begin{aligned} b_0 &= \cos(r_1) && (23) \\ b_1 &= \cos(r_2)(1 + \cos(r_2)). && (24) \end{aligned}$$

If (22) is made equal to the time-invariant allpass in (3), coefficients r_1 and r_2 can be expressed in terms of c and d,

$$\begin{aligned} r_1 &= \arccos(-c) && (25) \\ r_2 &= \arccos(-d). && (26) \end{aligned}$$

and thus in terms of the corresponding desired control parameters f_π and f_b as given by (4).

4.1 Differences in Output

Figures 3-6 show the difference in output between the original difference equation version of the filter and the new power-preserving one. Figure 3 shows the effect of an instanta-

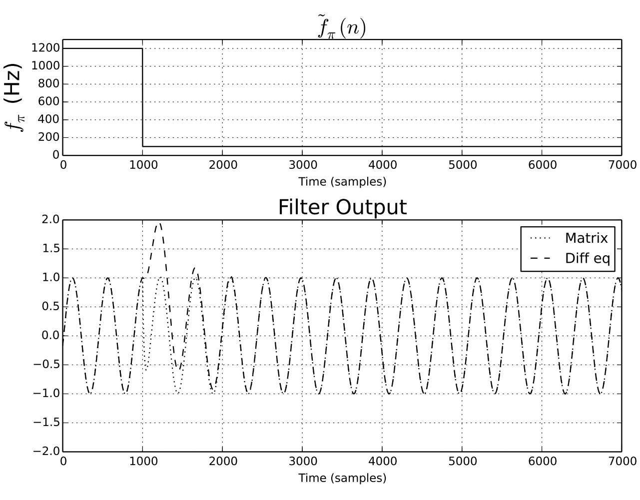

Figure 3. Effects of instantaneous change in f_π for difference equation and power-preserving filter types.

neous change in f_π on both filters. The difference equation form experiences a large jump in amplitude at the time where f_π changes, while the power-preserving form experiences a jump in phase. Figure 4 shows the difference in output between the two filters when they are made time-varying with the same sinusoidal modulation parameters. For the most part, the output is similar, but the difference equation form experiences larger spikes in amplitude. These spikes are much smaller in the power-preserving output. As implied by the waveforms in Figure 4, the output of the two filters is very similar in terms of harmonic content. In fact, as evidenced by

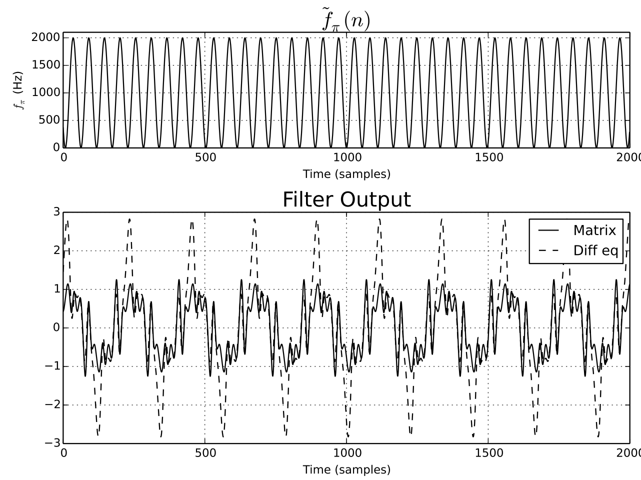

Figure 4. Output of difference equation and power-preserving filter types for identical modulation parameters. Parameters were chosen so that both filters would remain stable.

the analysis in Figure 5, the outputs of the two filters share the same harmonics, only differing slightly in their amplitudes. Figure 6 shows the output of both filter types with parame-

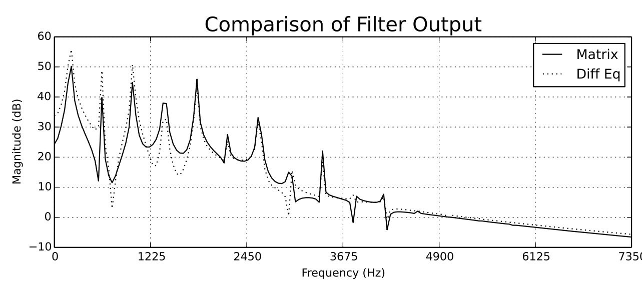

Figure 5. Magnitude spectrum of output of difference equation and power-preserving filter types for identical modulation parameters. Parameters were chosen so that difference equation filter would become unstable.

ters chosen so that the difference-equation form will become unstable. The power-preserving matrix form maintains stability, while the difference equation form produces output which grows rapidly.

5. ALLPASS FILTERS IN FEEDBACK NETWORKS

As discussed in the introduction, the aim of this project is to investigate the use of allpass filters in generative audio systems. It is thus useful to look at the behaviour and potential

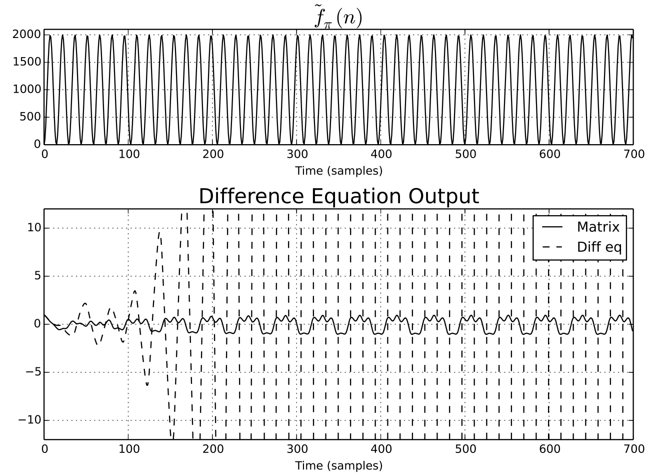

Figure 6. Output of difference equation and power-preserving filter types for identical modulation parameters. Parameters were chosen so that difference equation filter would become unstable.

sound production of various configurations of allpass filters in unity gain feedback networks. First, the time-invariant transfer function is examined, followed by a qualitative study of the spectra generated by the corresponding time-varying system. Finally, an application will be given, in which time-varying allpass filter is used to create a generative oscillator.

5.1 Transfer Function Analysis of Time-invariant Feedback Systems

In the examples in this section, if the systems are excited with an impulse, they behave as oscillators and may be left to oscillate indefinitely. The aim of this section is to gain insight into the character of the generative oscillator, as well as how they respond to control parameters f_π and f_b.

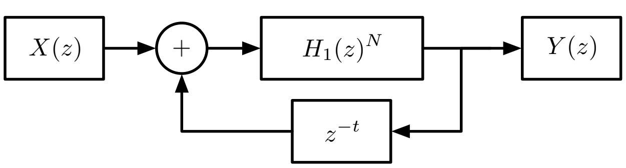

Figure 7. Block diagram of allpass filter cascade in feedback configuration.

The output of a feedback system, like the one shown in Figure 7, comprised of a cascade of N second-order allpass filters given by (29) and a feedback delay of T, may be expressed

$$Y(z) = \left(X(z) + Y(z)z^{-T}\right) H_1(z)^N, \tag{27}$$

yielding a system transfer function of

$$H_2(z,N) = \frac{Y(z)}{X(z)} = \frac{H_1^N(z)}{1 - H_1^N(z)z^{-T}}. \tag{28}$$

where

$$H_1(z) = \frac{b_0 - b_1 z^{-1} + z^{-2}}{1 - b_1 z^{-1} + b_0 z^{-2}}, \tag{29}$$

is the transfer function of the power-preserving second-order allpass filter whose difference equation is given in (3).

It is clear that since $H_2(z,N)$ is not an allpass filter, the overall effects of control parameters f_π and f_b are no longer what they were for $H_1(z)$ and warrant further exploration of their behaviour in their new context.

When $N = 1$ and $T = 1$, the system transfer function reduces to

$$H_2(z,1) = \frac{b_0 - b_1 z^{-1} + z^{-2}}{1 - (2b_0 + b_1)z^{-1} + (2b_0 + b_1)z^{-2} - z^{-3}}. \tag{30}$$

Representing H_2 in this way has the advantage of allowing observation of its poles and zeros (shown in Figure 8 for f_π constant at 11025 Hz and various values of f_b between 500 and 4000 Hz). As might be expected of a unitary feedback system, all three poles - two complex and one real (at DC) - are directly on the unit circle. A decrease in f_b corresponds to a movement of the two zeros downward toward the unit circle, and the two complex poles along the unit circle toward a point of pole-zero convergence.

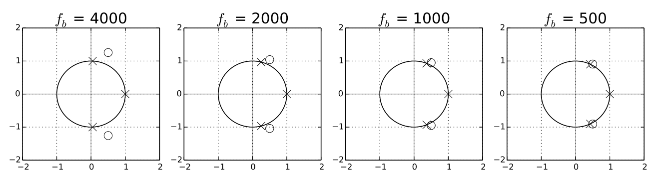

Figure 8. Example pole-zero plots as f_b is varied while f_π is kept fixed, demonstrating effect of f_b on zero location.

The behaviour of the poles and zeros has the effect, shown in Figure 9, of a resonant peak in the amplitude response being shifted somewhat downward in frequency, with a notable decrease in bandwidth. Accordingly, as shown in Figure 10, keeping f_b constant while increasing f_π results in a magnitude response characterized by a peak that loosely follows f_π, but with a bandwidth that decreases slightly with an increase in frequency. It is clear, therefore, that both parameters affect the position and width of the peak, with the bandwidth being more significantly influenced by f_b, and the position (frequency) more significantly influenced by f_π.

Finally, cascading multiple allpass filters (increasing N) has the effect of increasing the order of H_2, resulting in the introduction of an additional peak. Thus, as shown in Figure 11, a cascade of N filters within H_2 results in an amplitude response with N peaks (excluding DC).

5.2 Spectra of Time-varying Feedback Systems

The following discussion considers the effect of various modulation parameters on the spectra of the output of a time-varying feedback system. The parameters are made time-varying as shown in Section 2.2.

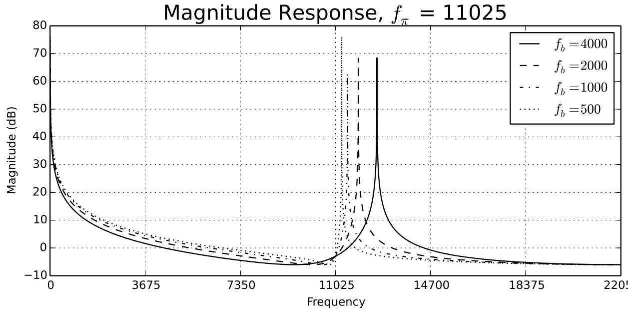

Figure 9. Example magnitude responses as f_b is varied while f_π is kept fixed, demonstrating decreasing peak location.

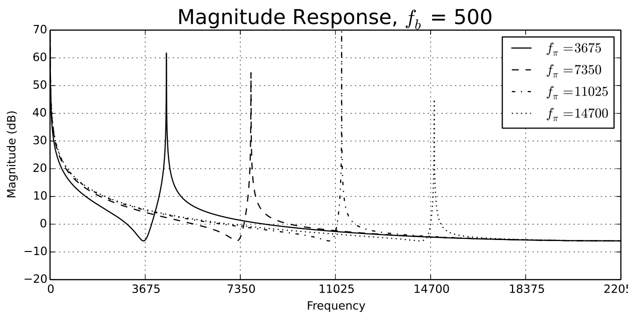

Figure 10. Example magnitude responses as f_π is varied while f_b is kept fixed, demonstrating increase in peak location.

The effect of each of the parameters f_π, f_b, f_m, and M on the output spectra is relatively clear, and in most cases is related to the effect on the non-feedback systems described in [6]:

- f_m - Controls spacing of sidebands. Sidebands are spaced at integer multiples of a central frequency (which depends on f_π and f_b as discussed above). Figure 12 shows this effect.

- f_π - Affects center frequency of sidebands. As f_π increases, the center frequency increases, though not in a $1:1$ relationship. Figure 13 shows this effect.

- f_b - Affects center frequency of sidebands. As f_b decreases, the center frequency decreases, becoming closer to f_π. Additionally, the bandwidth of the output is increased slightly with increasing f_b. Figure 14 shows this effect.

- M - Affects bandwidth of output. As M increases, bandwidth increases. Figure 14 shows this effect. Sidebands can alias around DC or the Nyquist frequency back into the spectrum.

Finally, as an important note, consider the sidebands present just above DC in Figures 12 - 15. These are components built around the DC term, introduced by the real pole of the feedback system.

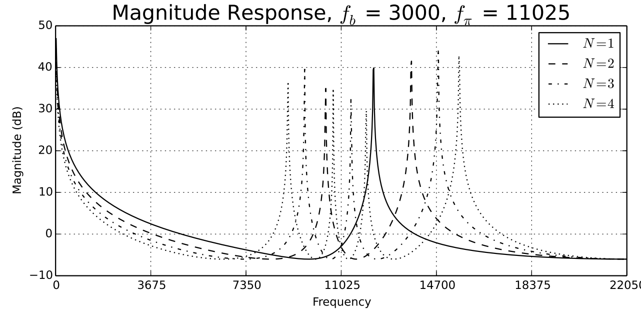

Figure 11. Example magnitude responses as N is varied while f_π and f_b are kept fixed, demonstrating centering around f_π and increasing flatness.

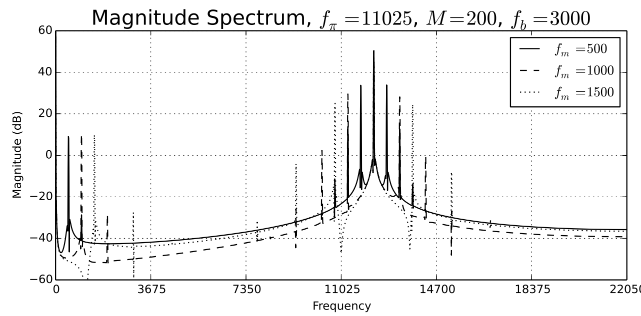

Figure 12. Output spectra of allpass feedback network with time-varying parameters. The parameter f_m changes, while the others are fixed.

5.3 A Self-Modulating Allpass Feedback Network

The final network investigated here is one in which the sinusoidal modulation of (5) is replaced with a modulation signal derived from a point in the network itself. In this configuration, similar to one posed in [10], the delayed output of the allpass filter is used to modulate the f_π parameter. A block diagram of this system is shown below in Figure 16.

In order to use such a configuration, the delayed output sample $y(n-1)$ must be scaled and biased in order to occupy the same range as the parameter to be modulated. For example, if we desire to modulate f_π with $y(n-1)$, then (5) is replaced with

$$\tilde{f}_\pi(n) = b + sy(n-1), \tag{31}$$

where s and b are user-defined scaling and bias factors, respectively.

In general, s and b should scale and offset $y(n-1)$ so that it falls into the range expected for $\tilde{f}_\pi(n)$. However, this is not strictly necessary for stability, as the cosine term in (7) will wrap any values into the range $-1 <= \tilde{d}(n) <= 1$. Thus, interesting results may be obtained by using other values for s and b.

5.3.1 Qualities of Musical Feedback Systems

A configuration like the one proposed in this section is very sensitive to particular combinations of s and b. It can therefore be difficult to assign meaningful parameters to such a

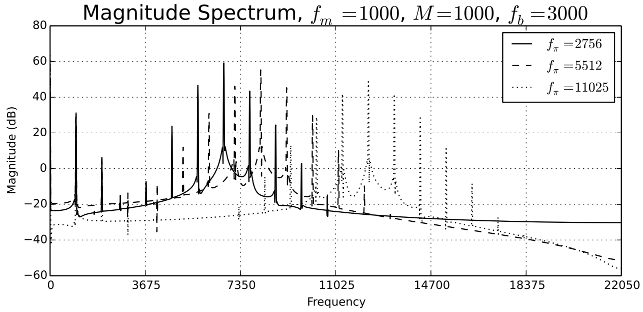

Figure 13. Output spectra of allpass feedback network with time-varying parameters. The parameter f_π changes, while the others are fixed.

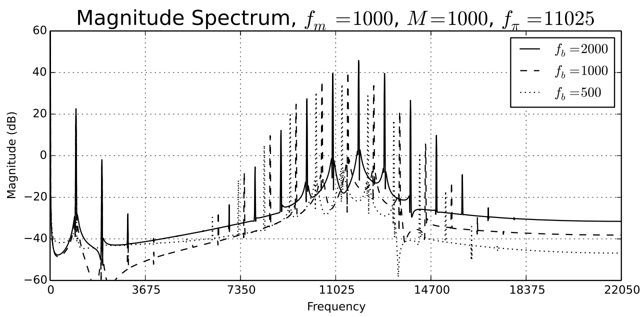

Figure 14. Output spectra of allpass feedback network with time-varying parameters. The bandwidth parameter f_b changes, while the others are fixed. $M = 1000$, $f_\pi = 1000$, $f_m = 1000$.

system. Sanfilippo et al. provide terminology which is useful in describing the behavior of musical feedback systems like this one [12]. First, the system is *iterative* - it is self-sustaining and produces variations on initial conditions. Second, there is *coupling* between components of the system. All components of the system are of equal importance, and this equality can lead to a specific set of characteristic behaviors. Finally, the system is also capable of *self-organization*, in that the output tends to oscillate between two or more distinct types of behaviors. Due to the *circularity* of the time-varying feedback system, in which "effects are also causes," there is also an interaction between different musical attributes - loudness, pitch, and timbre are interrelated.

This interrelatedness is easy to see when considering the feedback system described here: as the magnitude of the system's output at $y(n)$ increases, this causes a greater range of values for $\tilde{f}_\pi(n)$ - equivalent to increasing M in the case of sinusoidal modulation - and therefore a greater output bandwidth. Higher frequencies at the output - related to both pitch and timbre - cause a faster rate of change in $y(n-1)$. This is equivalent to increasing f_m and causes wider harmonic spacing around spectral components.

Practically, these systems often produce very high-amplitude output, which often has a DC offset, due to the unit-radius pole at DC. Therefore, it is often necessarily to scale and apply a highpass or DC-blocking filter to the output.

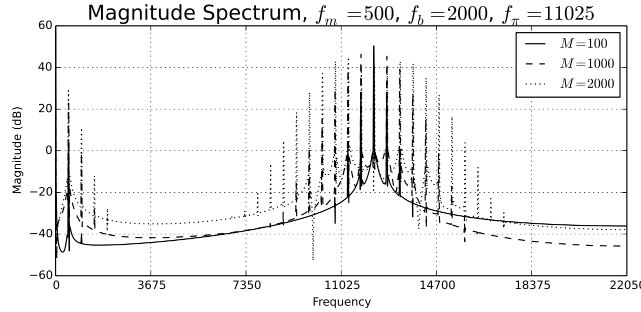

Figure 15. Output spectra of allpass feedback network with time-varying parameters. The parameter M changes, while the others are fixed.

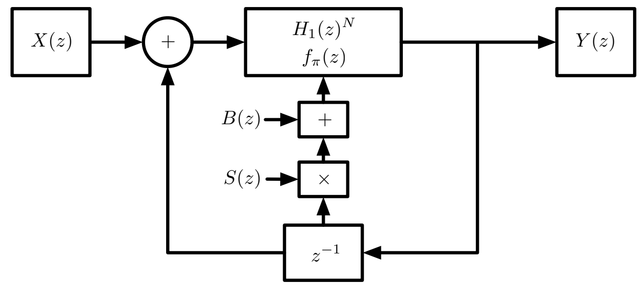

Figure 16. A high-level block diagram of a self-modulating allpass feedback network.

5.3.2 Sample Output

Consider a system where $\tilde{f}_\pi(n)$ is defined as

$$\tilde{f}_\pi(n) = 3333 + 1173y(n-1). \tag{32}$$

An example of the output of this system is shown in Figure 17. The characteristic output of this system consists of rhythmic "chirps" of increasing frequency, alternating with steady tones and short bursts of noise. The chirps are grouped into perceptible units by their duration. Most of the steady tones are short, such as those from 0.0" to 1.5", however others are longer, like those beginning at about 1.9". The rhythms are quasi-periodic, never repeating exactly - the sense is more of a series of iterative variations, rather than repetitions. Occasional interjections of noisy, modulated material occur, further challenging the sense of periodicity. There is a semi-periodic oscillation between three types of material: the rapid chirps, the steady tones, and the noise. Depending on the particular values of s and b, this self-organizing behavior can continue indefinitely or eventually settle into a more repetitive mode. It should be noted that the system is extremely dependent on initial conditions, and a given set of s and b will not necessarily produce the exact same results each time they are recalled.

Figure 18 shows another view of the characteristic behavior of this system. The values of f_b and b are the same as those used in Figure 17, while s has changed slightly (from 1173 to 1167). The change to s affects the rhythmic behavior of the system, as evidenced by the much shorter duration

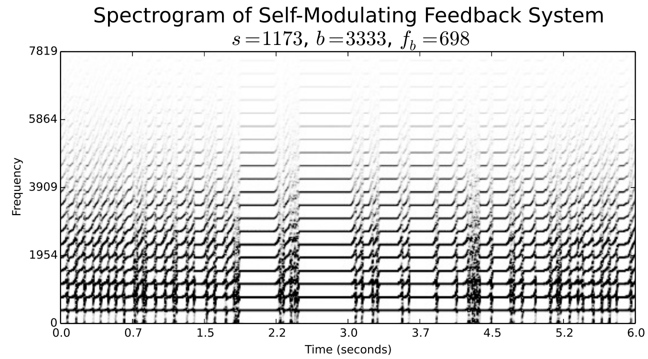

Figure 17. Spectrogram showing self-organizing behavior of self-modulating feedback system.

of the steady tones, and the increased prominence of noise. The harmonic structure changes as well, with the presence of more widely-spaced bands of emphasis (two are visible at approximately 800 and 2000 Hz), instead of the tightly spaced, clearly defined harmonics in Figure 17.

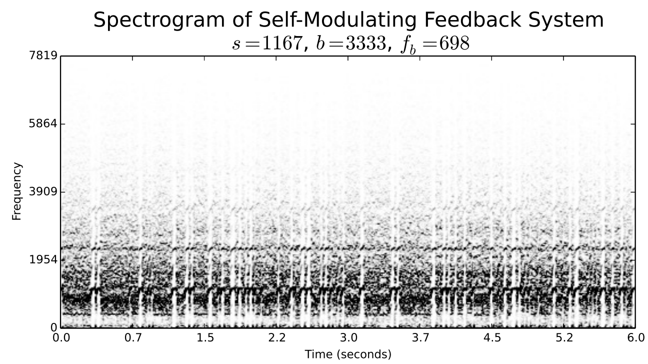

Figure 18. Spectrogram showing self-organizing behavior of self-modulating feedback system.

6. CONCLUSION

As shown, time-varying allpass filters have a variety of applications in signal processing and generative music. This paper described issues of stability in time-varying allpass filters.

Unfortunately, commonly used allpass filter formulations are susceptible to instability when their coefficients are made time-varying. Two methods of showing this instability were used, though neither is useful in a real-time situation. Instead, a power-preserving matrix form of the second-order allpass was presented. This version of the filter is stable when its coefficients are modulated.

The filter was used to extend a technique for applying frequency-selective phase distortion to signals, by incorporating a time-varying allpass filter into a unity gain feedback network. These networks were studied in terms of their static frequency response and their output spectra when the allpass component was made time-varying.

Finally, an application was given which demonstrated the potential usefulness of the power-preserving allpass filter in a generative audio system. A self-modulating feedback network was designed, in which a system's parameters were modulated by its own output. This type of system is capable of interesting, generative behavior, and will be studied further in larger compositional contexts. Possible expansions include more complex modulation schemes (for example. modulating s or b with signals from points in the feedback network), or building feedback networks using multiple allpass filters (or cascades) with differing modulation parameters.

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