CONVOLUTIONAL SYNTHESIS OF WIND INSTRUMENTS

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ABSTRACT

In this work we propose a new physical modeling technique whereby a waveguide structure is replaced by a low latency convolution operation with an impulse response that is either measured, modified, and/or constructed, optionally parametrically. By doing so, there is not longer the constraint that successive arrivals be uniformly spaced, nor need they decay exponentially as they must in a waveguide structure. Measured impulse responses allow for the estimation of filter transfer functions normally seen in a waveguide model of an acoustic tube, some of which are difficult to obtain theoretically, which may then be used to synthesize new impulse responses corresponding to wind instrument bores both existing and imagined. The technique is presented in the context of reed-based wind instruments, but may be extended to other musical instruments.

1. INTRODUCTION

Blowing into the mouthpiece of a reed-based wind instrument allows the player to control the reed's oscillation by creating a pressure difference across its surface. An increase in both upstream mouth pressure and downstream bore will however, cause the reed to either open or close further, depending on its configuration [1] and thus both pressures are important contributions to the behaviour of the reed. The oscillation of reed is therefore also dependent on the nature of the bore to which it is coupled, as it imparts frequency-dependent losses on the travelling pressure waves contained within, that are dependent on the bore's length, size, shape and termination.

A reed model must be coupled to a bore model to obtain a value for its downstream pressure. It is common to use waveguide synthesis to model the left and right travelling pressure waves, as it is practical, efficient, and produces very good results [2]. The waveguide model must, however, make certain approximations, particularly for the reflection at an open end termination, which is more difficult to account for theoretically. The model also becomes more complex, and not easily approximated by a one-dimensional waveguide structure, if the shape of the bore departs from a cylindrical or conical shape. In fact, any attempt to free the bore model of physical constraints which yield its harmonic resonant modes, is made more difficult using a one-dimensional waveguide model.

In this work we propose a new physical modeling technique whereby a waveguide structure is replaced by a low latency convolution operation with an impulse response that is either measured, modified, and/or parametrically constructed. The output of an excitation model, such as a reed, may be convolved with the impulse response in such a way as to permit very low-latency, enabling the modification of the impulse response in realtime, in ways both similar to, and different from, its waveguide counterpart.

The term *Convolutional Synthesis* from the title, borrows from *Convolutional Reverb*, a technique used for simulating reverberation of a physical or virtual space, using its characteristic impulse response. In this work, measurements of acoustic tubes, and musical instrument bores, are used as a base from which a parametric impulse response may be synthesized, corresponding to possible wind instrument bores, either existing, or imagined.

2. BORE PRESSURE IN A REED CONTEXT

It is by considering both the force F driving the reed, and the air flowing through the reed U, that the dependence of reed oscillation on bore pressure is apparent. The displacement of an oscillating wind-instrument reed may be approximated by the second-order differential equation

$$m\frac{d^2x}{dt^2} + m2\gamma\frac{dx}{dt} + k(x - x_0) = F, \qquad (1)$$

where m is the effective mass of the reed, γ is the damping coefficient, k is the stiffness of the reed. The overall driving force is equal to the sum of all forces acting on the reed, $F = F_m + F_b + F_U$, where the force F_m acts on the reed surface area seen by the mouth pressure p_m, the force F_b

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acts on the reed surface area seen by the bore pressure p_b, and the force F_U is applied by the flow U, forcing the reed open.

The differential equation governing air flow through the valve, fully derived in [1], is also dependent on bore pressure and is given by

$$\frac{dU}{dt} = (p_m - p_b)\frac{A(t_0)}{\mu\rho} - \frac{U(t_0)^2}{2\mu A(t_0) + U(t_0)T}, \qquad (2)$$

where $A(t)$ is the cross sectional area of the valve channel, and μ is the length of reed that sees the flow.

A reed may therefore be modeled digitally by obtaining a value every sample period for the displacement of the reed x, the flow U, and the pressure at the base of the bore p_b, in response to an applied mouth pressure p_m. An example model is shown in Figure 1 for the case of the clarinet.

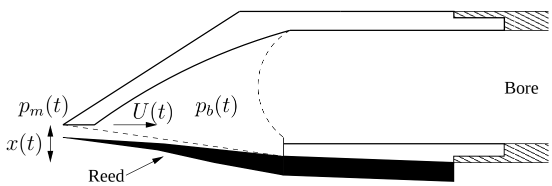

Figure 1: *A clarinet model showing the variables which evolve over time in response to an applied pressure $p_m(t)$.*

The pressure at the base of the bore, the reed's effective downstream pressure, is the sum of right and left travelling pressure waves at that position. The right traveling wave consists of the bore input pressure (the product of the characteristic impedance and the flow, Z_0U) along with any reflection off the reed termination, $R_cl(\omega)$ in Figure 2. The left traveling wave at this position consists of the incoming pressure wave returning from the bore and bell (before being reflected off the reed). The impulse response of a cylindrical bore, both rigidly closed and open, driven and measured at one end, may be seen in Figure 5 (bottom). An impulse response of a trumpet, both driven and recorded at the mouthpiece, may be seen in Figure 3.

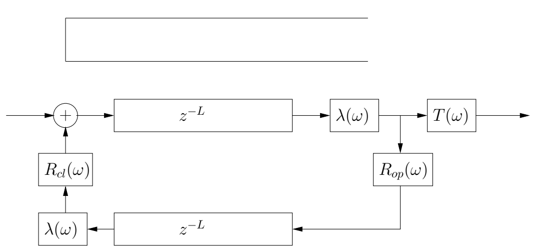

Figure 2: *A waveguide model of an open cylinder. If rigidly terminated, $R_{cl}(\omega) = 1$.*

It is common to model the pressure waves travelling along one dimension using a waveguide structure similar to the one seen in Figure 2. A change in the bore's cross sectional area, including the open-end termination, will impart a filtering caused by a reflection $R(\omega)$ and corresponding complimentary transmission $T(\omega)$. During propagation the waves are also subject to wall losses, with transfer function $\lambda(\omega)$ dependent on the size and length of the bore. The various ways in which the pressure waves are filtered, including the pure delay, constitute the model's waveguide elements, and have an observable effect on the resulting impulse response.

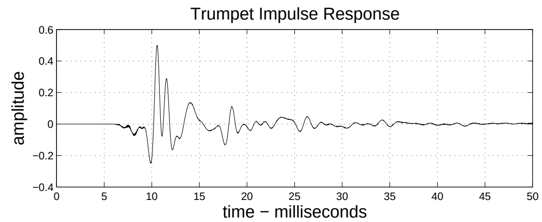

Figure 3: *An impulse response of the trumpet both driven and recorded at the mouth piece (i.e., not with the attached tube as shown in Figure 6).*

3. BORE RESPONSE MEASUREMENTS

There are certain approximations made by waveguide elements, such as the reflection occurring at an open end, a function of the complex terminating impedance $Z_L(\omega)$ which is a fairly complicated function of frequency. One-dimensional waveguide models can only be used to model cylinder or conical tubes and any departure from these shape will add a complexity that may be difficult to account for theoretically (though again, certain approximations may be made). We therefore use a low latency convolution with an impulse response, either fully measured, modified, or constructed, as an alternative to the waveguide model.

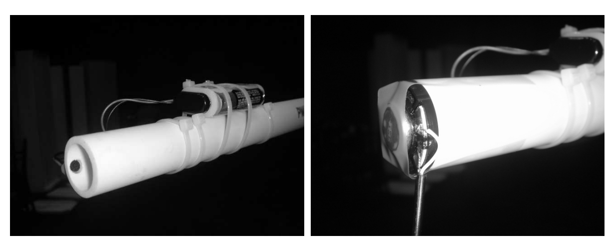

Figure 4: *A microphone and co-located speaker.*

It is well known that inputting an impulse into an LTI system yields its impulse response. Here, we use a sine swept over a frequency trajectory, effectively smearing a sufficiently loud impulse over a period of time so that the system may be adequately driven without distortion [3]. As shown in Figure 4, the signal drives the tube via a speaker placed at one end, closing that end with a termination given by $R_cl(\omega)$, while a co-located microphone records the response, a measurement of the pressure at the base of the bore p_b.

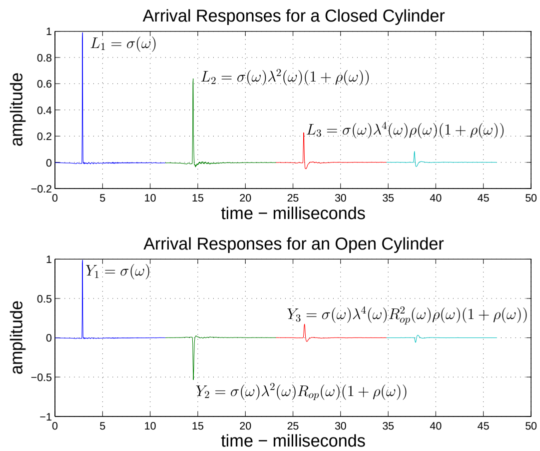

Figure 5: *Arrival responses are shown, with their corresponding transfer functions, for a cylinder that is rigidly terminated (top) and open (bottom).*

The impulse response for a two meter long cylinder, with the end opposite the speaker both rigidly terminated and open, is shown on the top and bottom of Figure 5, respectively. Within each impulse response there is a sequence of individual arrivals, L_n for the rigidly terminated and Y_n for the open tube, each one corresponding to a round-trip propagation of pressure waves, from the speaker to the co-located microphone. Each individual response contains different filtering, as responses appearing later in the sequence have been circulating in the tube for longer. The tube is first rigidly terminated (ensuring a perfect reflection) so the speaker transfer function $\sigma(\omega)$, and the reflection off the speaker $\rho(\omega)$, may be estimated and the system calibrated. The tube was then opened so the reflection function from an open end $R_{op}(\omega)$ could be estimated. Each measurement produced data that very closely matched the output of the waveguide structure in 2, with $R_cl(\omega)$ set to the speaker reflection $\rho(\omega)$.

In [4] the tube is also terminated with a conical section enabling the estimation of its reflection function. These results were also shown to correspond closely with theoretical expectation, thus providing the confidence that the reflection functions of various instrument bores could also be accurately estimated by appending the instrument to the calibration tube, as shown in Figure 6, and then comparing the response to that of the calibration tube when it is rigidly terminated.

Figure 6: *Measuring the clarinet and trumpet to obtain their reflection functions.*

4. CONVOLUTIONAL SYNTHESIS

The notion of replacing the reflection and transmission functions terminating a waveguide structure (such as the one seen in Figure 2) with a measured reflection function is not new. In [5], the output of a pure delay, the length of which is dependent on the desired pitch, is convolved with a measured reflection function, corresponding to a round-trip propagation of pressure from the speaker to the microphone. Leading zeros are replaced with a delay line of optional length to allow for control of pitch.

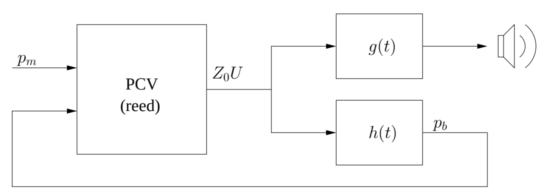

Figure 7: *A signal flow diagram of convolutional waveguide synthesis in the context of a reed instrument. The bore pressure p_b is obtained by convolving the bore input pressure Z_0U with the impulse response $h(t)$, and the model output is obtained by a convolution with the impulse response $g(t)$.*

Here we propose substituting the entire waveguide structure, as seen in Figure 2, with a low latency convolution with the complete impulse response, shown in Figure 7. In this case, the reed produces an input pressure, a product of the characteristic impedance and the flow Z_0U, which is convolved with the impulse response $g(t)$ from the reed (or speaker location) to outside the bell, to obtain the output sound, and then convolved with the impulse response $h(t)$ from the speaker to the co-located microphone, to obtain the bore pressure p_b. It should be noted that $g(t)$ may be inferred from $h(t)$ with the assumption that a transmission is always complimentary to its corresponding reflection; it is not necessary to make a separate measurement unless there is a different desired listening location than just outside the bell.

Given the impulse response of an acoustic tube structure, a low latency convolution operation, as described in [6], with the pressure due to flow from the reed model Z_0U from Figure 7, would yield the same level of interactive control as the waveguide model. The user has the ability to modifying the impulse response every blocksize of samples, where the blocksize is significantly smaller than the entire length of the impulse response. If the impulse response is constructed, or synthesized, parametrically, the user may therefore modify the parameters in realtime.

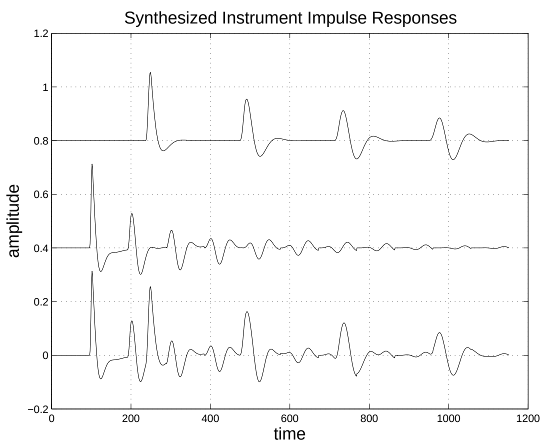

Figure 8: *An impulse response is synthesized by interleaving two separate impulse response (top and middle) to produce a multi-phonic impulse response (bottom).*

The structure of the simplest cylindrical tube impulse response as shown in Figure 5, is rather straightforward to synthesize: an initial impulse followed by additional arrivals uniformly spaced, each one decaying exponentially and containing varying filtering depending on its position in the sequence. One parameter that may be made available to the user in this case is the spacing between the arrivals, which may be stretched or contracted depending on the desired pitch. It is also possible to use an entire impulse response from an instrument, such as the one shown for a trumpet in Figure 3. In this case the reflection $R_{cl}(\omega)$ from Figure 2 depends on the opening of the reed, a filtering likely more significant for lip reeds than for clarinet reeds, and the impulse response should be modified to account for this. This is not done for Figure 3, though we have a technique in mind not yet implemented.

At the expense of computational complexity, the low latency convolution affords the user some additional advantages to the waveguide model. For example, it is possible to build impulse responses less typical of a physical system, which would be difficult to obtain within a waveguide structure. There is no longer the physical constraint of having echo responses uniformly spaced. Rather, echos may be interleaved in patterns that create multiple tones (as opposed to a single fundamental frequency), creating a sort of multiphonic instrument (see Figure 8). The echos are also no longer constrained to decay exponentially; they may grow and then suddenly drop, with rates specified parametrically if so desired.

5. CONCLUSIONS

We propose a physical modeling synthesis that replaces waveguide structures with a convolution with the entire impulse response (not just the impulse response of the reflection at the bell), allowing for the possibility of manipulating the entire impulse response to produce interesting effects. Using a complete impulse response makes it possible to modify the spacing between successive arrivals without the physical constraint of them being uniformly spaced. The individual arrivals may also be modified so as to change their gain, or even their constituent transfer functions, allowing for a level of response processing that sets it rather apart from the waveguide model. The convolutional synthesis technique requires the use of a low latency implementation of the convolution operation so the model can run in real time, while allowing for real-time parameter manipulation of both the reed and bore models.

6. REFERENCES

[1] T. Smyth, J. Abel, and J. O. Smith, “The feathered clarinet reed,” in *Proceedings of the International Conference on Digital Audio Effects (DAFx’04)*, Naples, Italy, October 2004, pp. 95–100.

[2] J. O. Smith, *Digital Waveguide Modeling of Musical Instruments*. www-ccrma.stanford.edu/˜jos/waveguide/, 2003.

[3] A. Farina, “Simultaneous measurement of impulse response and distortion with a swept-sine technique,” in *Proceedings of the 108th AES Convention*, Paris, France, February 2000, pp. 18–22.

[4] T. Smyth and J. Abel, “Modeling and measurement of wind instrument bores,” in *19th International Congress on Acoustics*, Madrid, Spain, September 2007.

[5] X. Rodet and C. Vergez, “Phyical models of trumpet-like instruments: Detailed behavior and model improvements,” in *Proceedings of ICMC 1996*. Clear Water Bay, Hong-Kong: International Computer Music Conference, August 1996.

[6] W. G. Gardner, “Efficient convolution without input-output delay,” *Journal of the Audio Engineering Society*, vol. 43, no. 3, pp. 127–136, March 1995.