Music 171: Sinusoids

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What is Sound?

- The word *sound* is used to describe both:

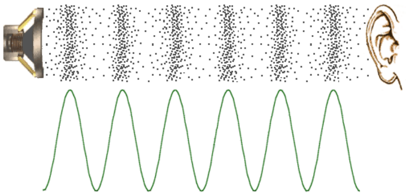
  1. an auditory sensation in the ear

  2. the disturbance in a medium that causes an auditory sensation

- Nearly all objects will vibrate when disturbed.
- Sound is a **wave** created by vibrating objects.
  – alternating regions of low and high pressure that propagate through a medium from one location to another.

Sound Wave

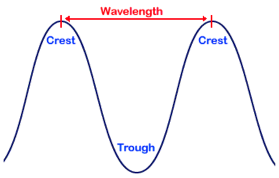
- Sound is a **mechanical wave**:
  – it requires a *medium* in which to propagate.
- Sound is a **traveling wave**:
  – it propagates from one location to another
- In fluids (air, water) sound is a **longitudinal** wave (has alternating regions of low and hight pressure).



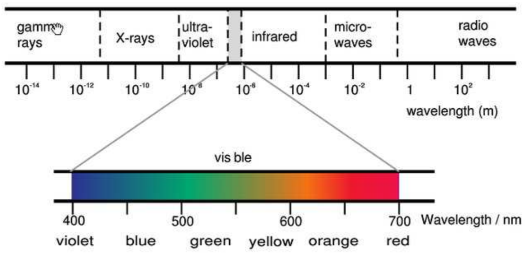
- The **waveform** illustrates the *pressure variation* (either temporal or spatial) of the wave.

Waveform: wavelength

- The **wavelength** is the *length* of one **cycle**:
  – distance between crests or between troughs



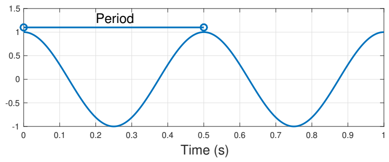
- Waves are often characterized by their **length**:



- **infrasonic**: lower than lowest audible frequency.
- **ultrasonic**: higher than highest audible frequency.

Waveform: period/frequency

- Waves may also be characterized by their period and/or frequency.
- The **period** is the *time* to complete one cycle:



- If we know wave velocity, we can determine the period from the wavelength:

$$\text{period} = \frac{\text{wavelength}}{\text{velocity}}$$

- The **frequency** is:
  - the number of cycles per second (Hz)
  - inverse of the period

$$\text{frequency} = \frac{1}{\text{period}} = \frac{\text{velocity}}{\text{wavelength}}$$

Waveform

- The *temporal waveform* of the sound shows the time evolution of the *pressure* variations, illustrating:
  - **amplitude**: maximum particle displacement from rest position (Pa).
  - **period**: time to complete one cycle (s).
  - **frequency**: number of cycles per second (Hz).
  - **wavelength**: length of one complete cylce (m) (need wave speed to compute).

Properties of Sound Waves

- Speed of sound[1]:
  - in air: 340 m/s
  - in water: 1480 m/s
- Amplitude range of hearing (humans)
  - Threshold of audibility: 0.00002 Pa
  - Threshold of feeling (or pain!): 200 Pa
- Frequency range of hearing
  - humans: 20 - 20 000 Hz
  - dogs: 20 - 45 000 Hz
  - beluga whale: 1000 - 123 000 Hz
- Period of lowest and highest audible frequencies
  - 1/20 Hz = 0.05 s   1/20 000 Hz = 0.05 ms
- Shortest human audible wave (air)
  - 340/20000=1.7cm
- Longest human audible wave (air)
  - 340/20=17m

[1] Quantity depends on temperature: For air, the speed of sound is $c = 20.1\sqrt{T}$, where T is the absolute temperature found by adding 273 to the temperature on the Celsius scale.

Why Sinusoids are Important

- Sinusoids are fundamental in physics: many systems oscillate in a quasi-sinusoidal motion known as **simple harmonic motion**:
  - repetitive movement through an equilibrium position
  - maximum displacement on either side of equilibrium is equal (if no losses).

- Sound is *bandlimited* and may be approximated by the sum of sinusoids (having frequencies up to a maximum value).

Sinusoids

- "Sinusoids" may be represented using both sine and cosine functions having the form:

$$x(t) = A\sin(\omega t + \phi) \quad \text{or} \quad x(t) = A\cos(\omega t + \phi),$$

where $x(t)$ is the quantity that varies over time and

$$\begin{aligned} A &\triangleq \text{peak amplitude} \\ \omega &\triangleq \text{radian frequency (rad/sec)} = 2\pi f \\ f &\triangleq \text{frequency (Hz)} \\ t &\triangleq \text{time (seconds)} \\ \phi &\triangleq \text{initial phase (radians)} \end{aligned}$$

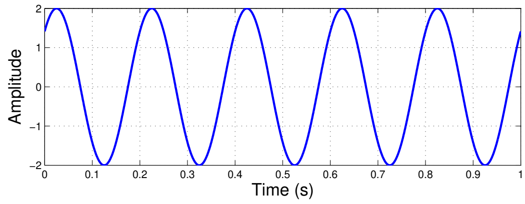
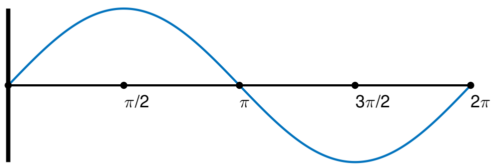


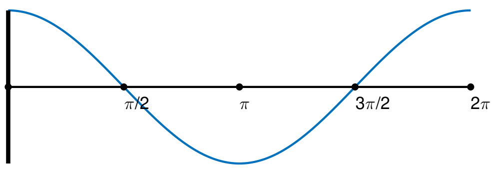
Figure 1: Sinusoid where $A = 2$, $\omega = 2\pi 5$, and $\phi = \pi/4$.

Cycle and (Initial) Phase

- One **cycle** of a sinusoid is 2π radians.
- The **phase** refers to the position within the cycle and is dependent on whether using a
  1. sine function ($\sin(\omega t)$):



  2. or a cosine function ($\cos(\omega t)$):

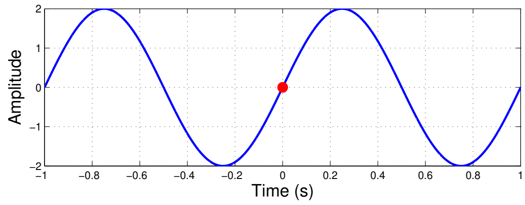


- The **initial phase** is the phase at time = 0 and is

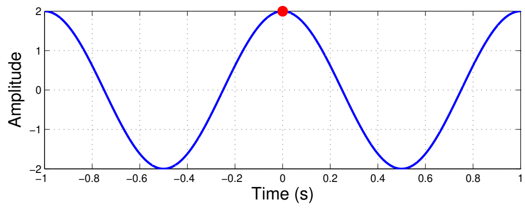
$$-\pi < \phi < \pi, \quad \text{or} \quad 0 < \phi < 2\pi.$$

Initial Phase

- The **initial phase** ϕ (or *phase offset*), in radians, gives the position of the waveform cycle at $t = 0$.
- Sine function with $\phi = 0$ ($\sin(\omega_0 t + 0)$):



- Sine function with $\phi = \pi/2$ ($\sin(\omega_0 t + \pi/2)$):



- ... or equivalently $\cos(\omega_0 t + 0)$.

Frequency (Hz vs. Rad/s)

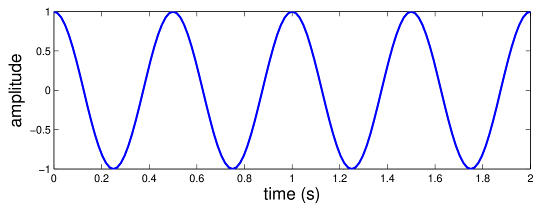
- The frequency f of the waveform is given in cycles per second or Hertz (Hz).
- Frequency is equivalent to the inverse of the period T of the waveform,

$$f = 1/T \quad \text{Hz}.$$

- The radian frequency ω, given in radians per second, is equivalent to the frequency in Hertz scaled by 2π,

$$\omega = 2\pi f \quad \text{(rad/sec)}.$$

- What is the frequency in Hz of the following sinusoid?



- What is the frequency in rad/sec?

Sine and Cosine Functions

- The sine and cosine function are very closely related and can be made equivalent simply by adjusting their initial phase:

$$\sin\theta = \cos(\theta - \frac{\pi}{2}) \qquad \text{or} \qquad \cos\theta = \sin(\theta + \frac{\pi}{2}).$$

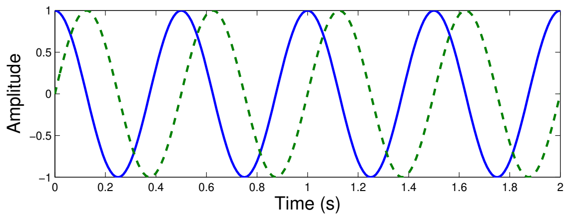


Figure 2: Phase relationship between cosine (solid blue line) and sine (broked green line) functions.

Amplitude Envelopes

- In his work, *On the Sensations of Tone*, Hermann von Helmholtz characterized tones by the way in which their amplitudes evolved over time, that is, by their *amplitude envelope*.
- He described the envelope as having three parts:
  1. **the attack**: the time it takes the sound to rise to its peak
  2. **the sustain**: the steady state portion of the sound (where the amplitude has negligiable chane)
  3. **the decay**: the time it take for the sound to decay or fade out.

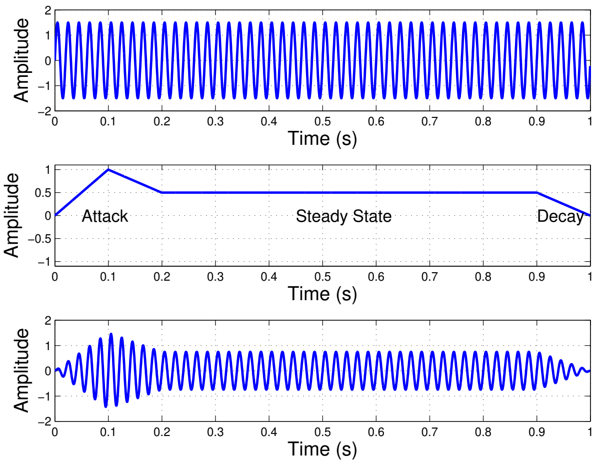
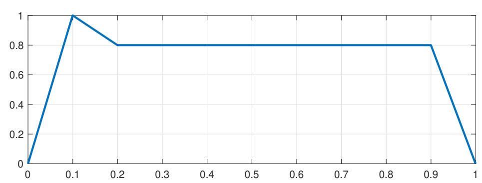


Figure 3: A sinusoid with an amplitude envelope.

ADSR Envelope

- The duration of the attack and decay greatly influence the quality of a tone:
  - wind instruments tend to have long attacks, while percussion instruments tend to have short attacks.
- Another envelope, called ADSR, has a fourth segment inserted between the attack and the sustain.
  - **A**: Attack
  - **D**: Decay
  - **S**: Sustain
  - **R**: Release



- The ADSR attempts to mimic envelopes found in musical instrument tones.