Music 175: Spectrum

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Adding Sinusoids at Different Frequencies

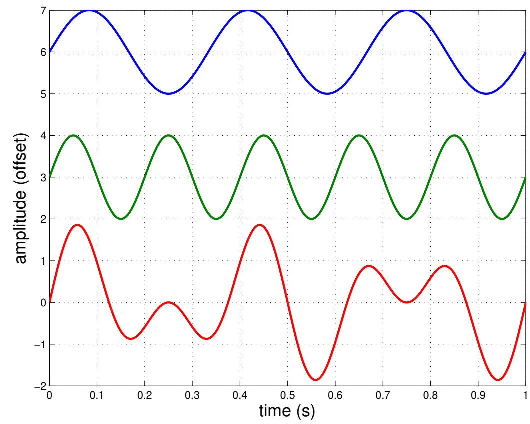


Figure 1: Adding 2 sinusoids at different frequencies.

- When adding sinusoids at different frequencies, the resulting signal is no longer sinusoidal.
- But is it (the visible wave) periodic?

A Non-Sinusoidal Periodic Signal

- If the frequencies of the added sinusoids are integer multiples of the fundamental, the resulting signal will be periodic.

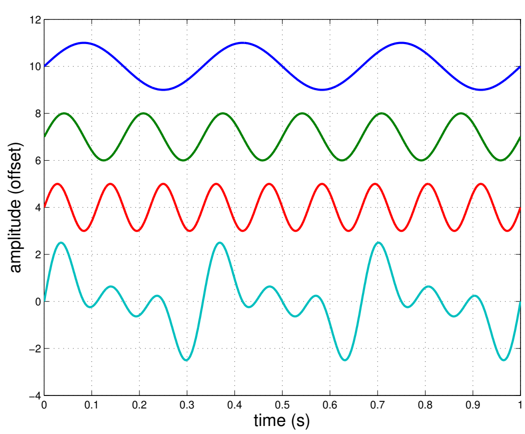


Figure 2: Adding sinusoids at 3, 6, 9 Hz produces a periodic signal at 3 Hz.

- Sinusoidal components, or *partials* that are integer multiples of a fundamental are calle *harmonics*.

Spectrum: Viewing in the Frequency Domain

- When viewing in the frequency domain, a “spike” at a frequency indicates there is a sinusoid in the signal at that frequency.

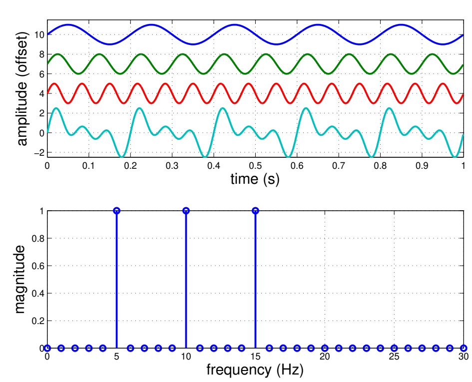


Figure 3: Adding sinusoids at 5, 10, 15 Hz in both time and frequency domain.

- The height of the spike indicates the amplitude—all sinusoids here have a peak amplitude of 1.
- The harmonic relationship between the partials can be seen by the even spacing between the “spikes”.

Standard Periodic Waveforms

- *square*, *triangle* and *sawtooth* by adding sinusoids.

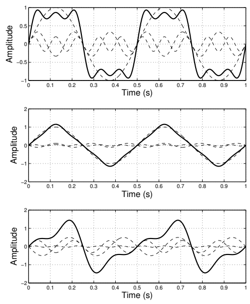


Table 1: Standard Waveforms Synthesized by Adding Sinusoids

Type	Harmonics	Amplitude	Phase (cos)	Phase (sin)
square	$n = [1, 3, 5, ..., N]$	$1/n$	$-\pi/2$	0
triangle	$n = [1, 3, 5, ..., N]$	$1/n^2$	0	$\pi/2$
sawtooth	$n = [1, 2, 3, ..., N]$	$1/n$	$-\pi/2$	0

Spectra of Standard Waveforms

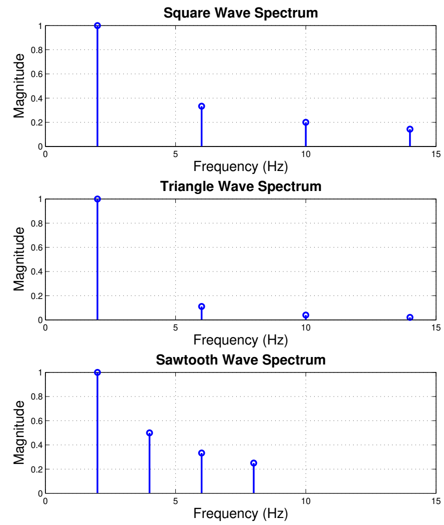
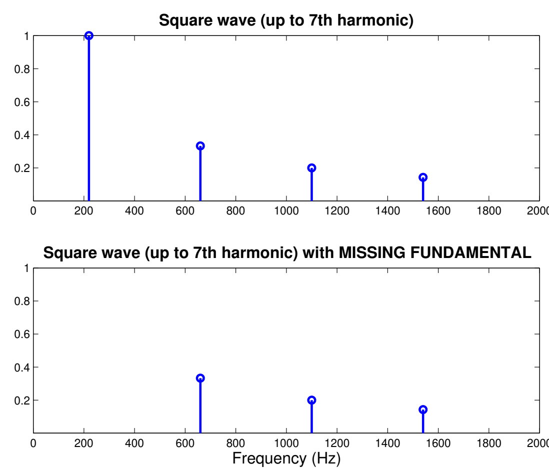


Figure 4: Spectra of complex waveforms

Harmonics and Pitch

- Notice that even though these new waveforms contain more than one frequency component, they are still periodic.
- Because each of these frequency components are **integer multiples** of some fundamental frequency, they are called *harmonics*.
- Signals with **harmonic spectra** have a fundamental frequency and therefore have a **periodic waveform** (the reverse is, of course, also true).
- **Pitch** is our subjective response to the **fundamental frequency**.
- The **relative amplitudes of the harmonics** contribute to the **timbre** of a sound, but do not necessarily alter the pitch.
- harmocity.pd

Missing Fundamental



- Listen to:
  - square wave: squaref0.wav:
  - square wave, NO fundamental: squareNOf0.wav:
- Does the sense of pitch change? How about timbre?

Inharmonicity

- Generally, inharmonic overtones lack a clear sense of pitch (difficult to hum).
- The *perception* of pitch
  - may vary with individuals;
  - tends to be clearer when notes are played in succession (particularly with inharmonic tones).
- Listen to:
  - bellsclip.wav: bell in isolation—pitch?
  - bells.wav: bells in melodic context—pitch?
- The context allows us to focus on the *change* in notes rather than on any one note itself.

Clarinet Analysis

- The (steady-state) tone of clarinet, mostly closed-open, is shown in time and frequency domain.

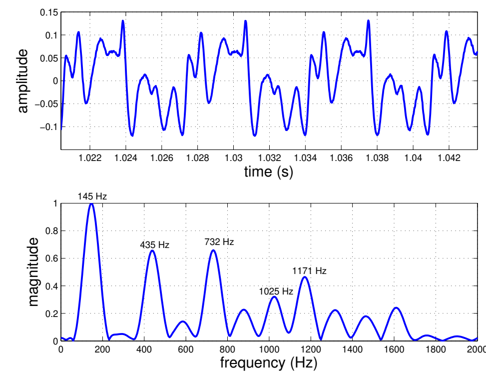
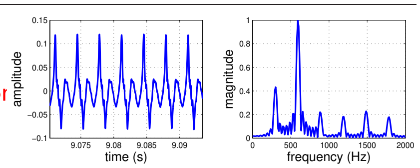


Figure 5: Frequency analysis of a clarinet note.

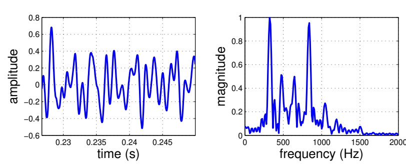
- Summing sinusoids at 145, 433, 732, ..., can approximate a steady-state synthesis of the clarinet:
  - clarOrigNormalized.ff.D3.wav
  - clarAttackRemoved.wav (steady state)
  - clarSynth.wav (single amplitude envelope)

Harmonicity and Pitch

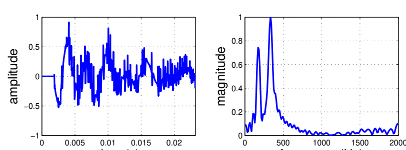
Tenor Saxophone:



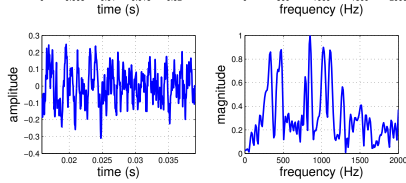
Japanese Bell:



Snare Drum:



Thunder:

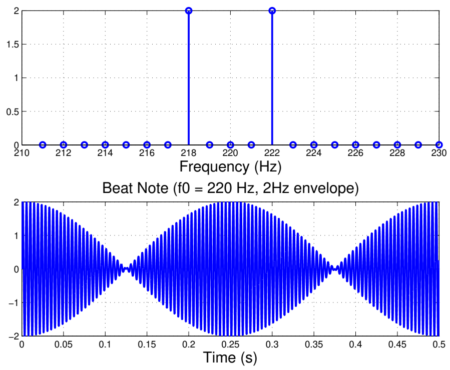


Pitch and Frequency

- Listeners usually compare tones on the basis of the musical interval separating them: m3, P5, P8 etc.
- **An octave (P8) corresponds to a doubling of frequency.**
- There is a nonlinear relationship between pitch perception and frequency in Hz:
  - an octave above 220 Hz is an increase of 220 Hz;
  - an octave above 440 Hz is an increase of 440 Hz.
- In equal-tempered tuning, there are 12 evenly spaced tones in an octave, called semi-tones:
  - The frequency n semitones above A440 is
    $$440 \times 2^{n/12} \text{ Hz.}$$
  - The frequency n semitones below A440 is
    $$440 \times 2^{-n/12} \text{ Hz.}$$
- pitchFreq.pd

Beat Notes

- What happens when we add two frequencies close in value?



- The waveform shows a periodic, **low frequency** amplitude envelope superimposed on a higher frequency sinusoid creating a beat note.
- This can be explained by the Cosine Product formula:

$$\cos(a)\cos(b) = \frac{\cos(a+b) + \cos(a-b)}{2}$$

- beat.pd