Music 171: Wavetables and Samplers

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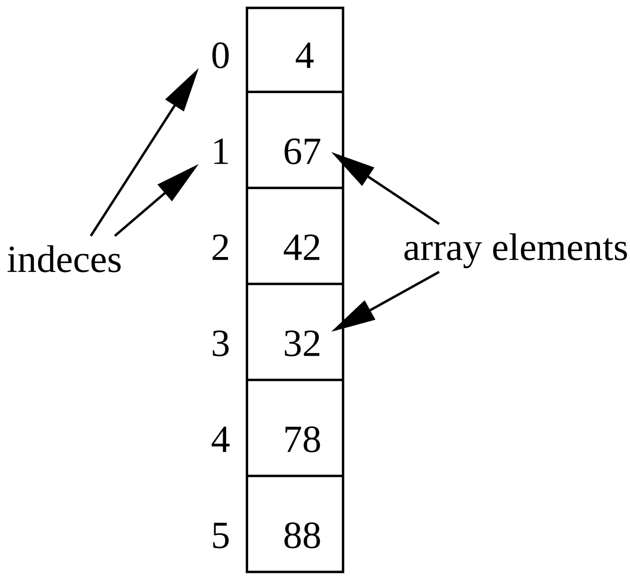
Array

- **Array**: a construct (data structure) that can be used to collect and organize sequences of numbers.
  - each array element (number) may be accessed by its **index**—its position in the array.
  - indeces typically begin with 0 and end with $N - 1$, where N is the length (number of elements) in the array.



- A **table** may be viewed as an array (an array may be used to implement a table).

Recall Sampling

- **Sampling**: process of taking a sample (value) of a continuous waveform at regular time intervals T_s.
- **Sampling rate**: frequency at which samples are taken:

$$f_s = \frac{1}{T_s} \text{ Hz.}$$

- Sampling the continuous-time sinusoid:

$$x(t) = A\sin(\omega t + \phi),$$

involves substituting continuous-time t with integer n multiples of the sampling period T_s:

$$t \longrightarrow nT_s$$

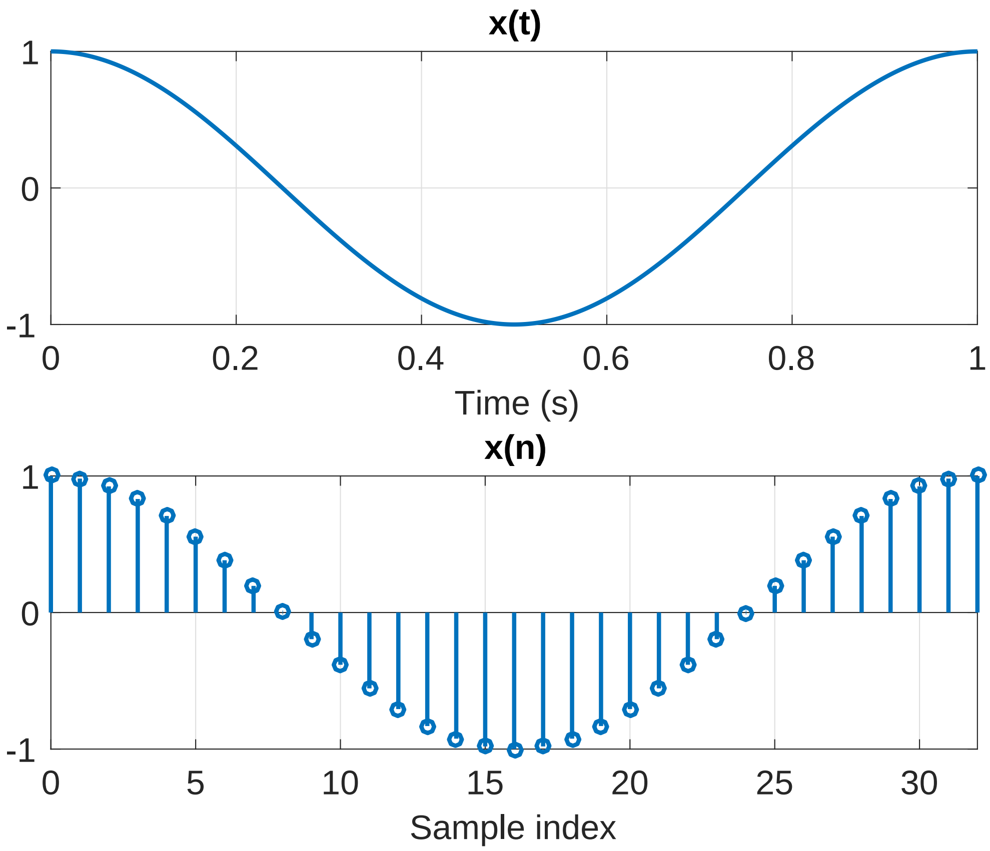
yielding a discrete-time sinusoid:

$$x(n) = A\sin(\omega n T_s + \phi).$$

- Integer n corresponds to the **index** of sequence $x(n)$.
- Sinusoid $x(n)$ may be implemented as an array or *wavetable*.

Recall Sampling and Reconstruction

- Once $x(t)$ is sampled to produce $x(n)$, *time scale information* is replaced with *sample index*:



- Sequence $x(n)$ may represent a number of sinusoids with **frequency** dependent on
  - **time between samples** or equivalently
  - **rate at which the table is read**.

Wavetable

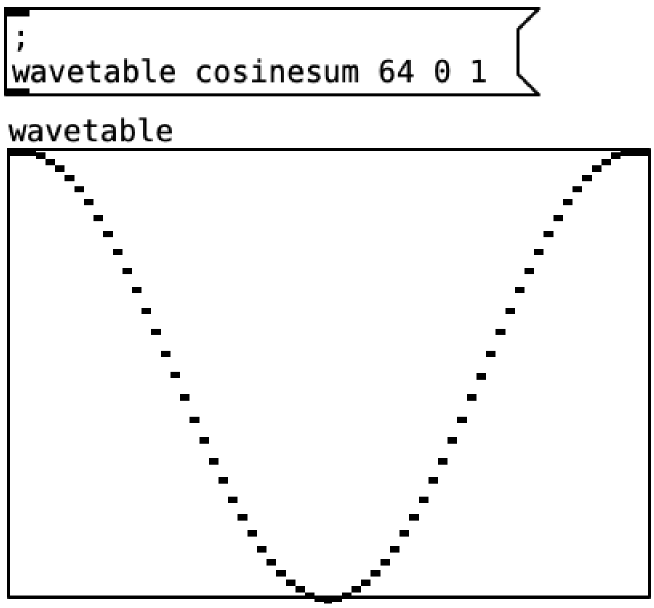
- A stored digital audio signal (e.g. sinusoid) is merely a sequence (or array) of N numbers:

$$x(n) \text{ for } n = 0, ..., N-1,$$

where n is the array index.

- Since a sinusoid is periodic, anything more than one period is, by definition, redundant.

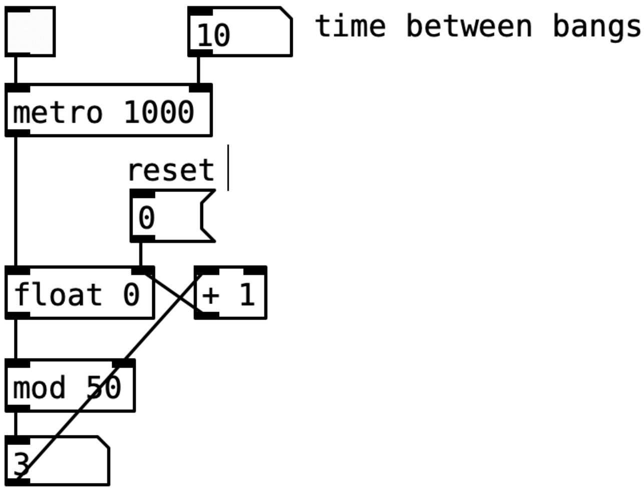
- **Store one period in a wavetable and read table at different rates.**



- How do we read from the table at different rates?

Wavetable Input Signal

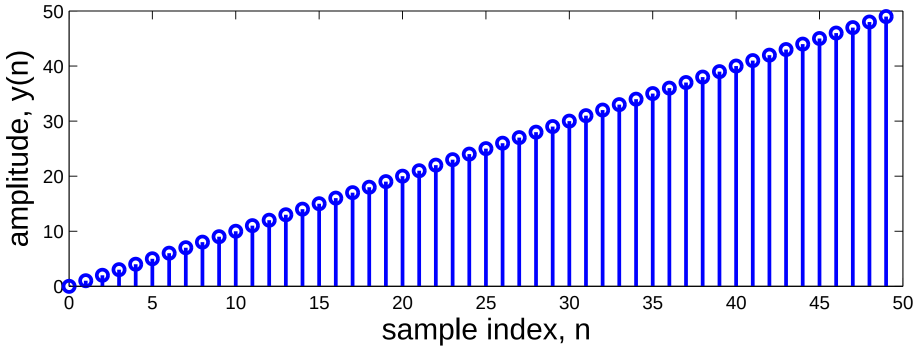
- To read from the wavetable from beginning to end, generate index values 0 to $N-1$.
- Can we use a counter?



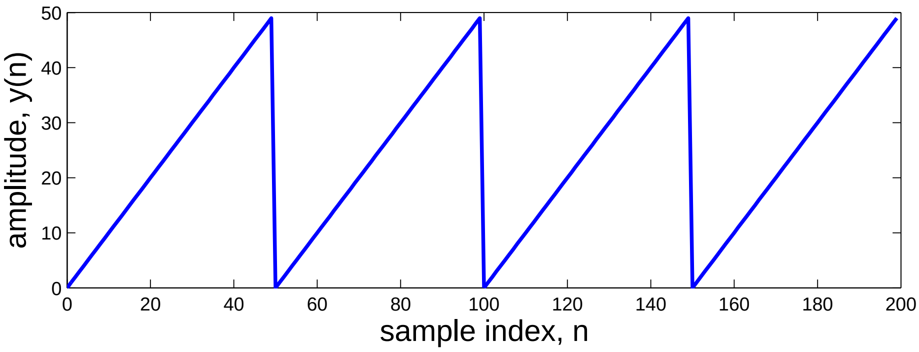
- We must generate index values at an **audio rate** and the counter produces values at a **control rate**.

Ramp Function and Sawtooth Waveform

- Consider a "ramp" function, having incremental values from 0 to $N-1$:

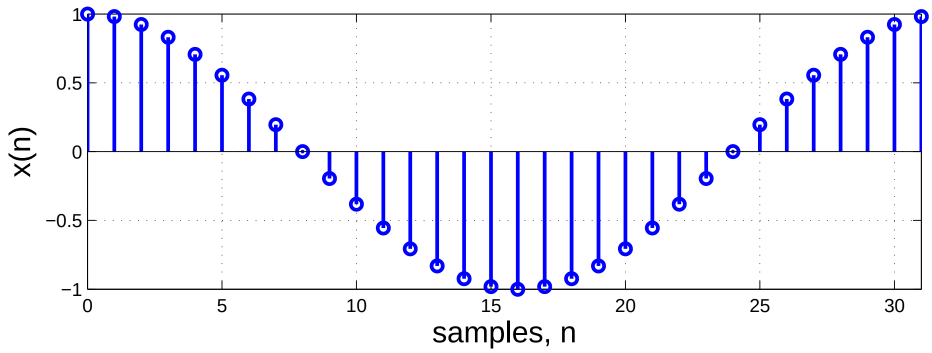


- Values played sequentially can be used as indeces to read the wavetable.
- To loop the wavetable (restart once ended), use a periodic ramp function (positive-valued **sawtooth wave**):

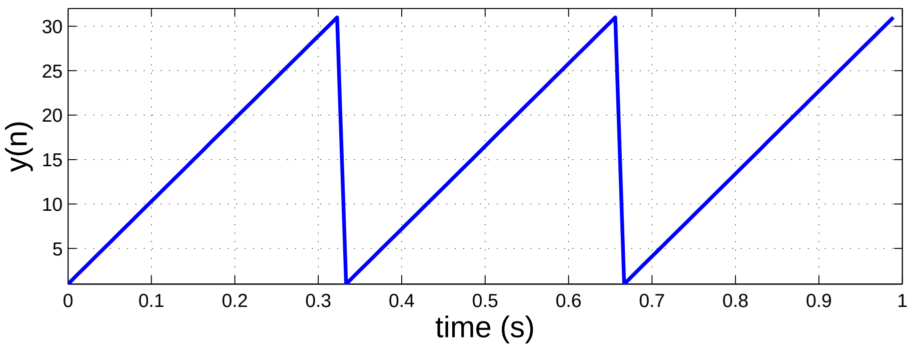


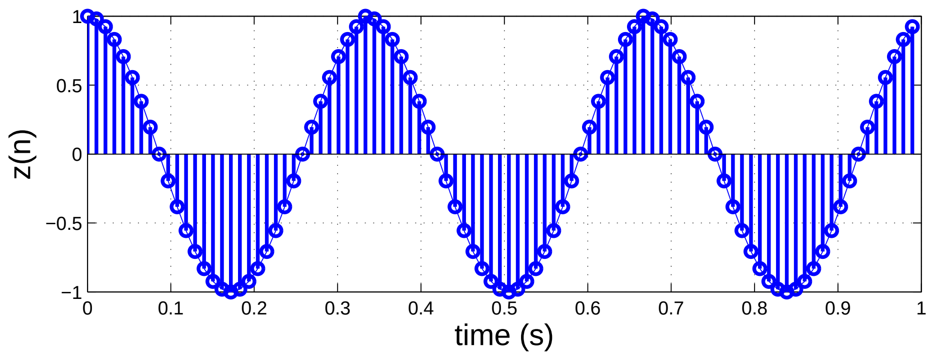
Wavetable Oscillator

- Consider wavetable $x(n)$ having one period (or cycle) of a sinusoid:



- To generate a 3-Hz sinusoid, read $x(n)$ 3 times per second by using a *3-Hz sawtooth wave*.



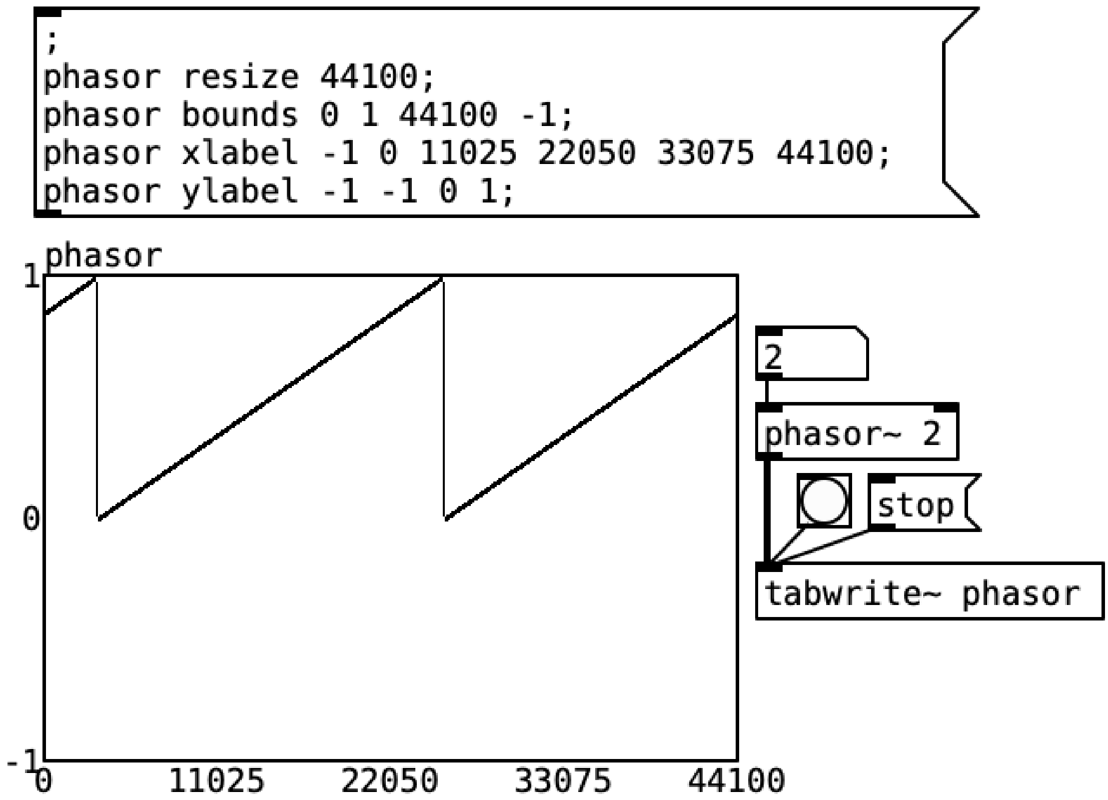


Wavetable Lookup

- Signal $y(n)$, a (positive-valued) sawtooth or *phasor*, when multiplied by $N - 1$, produces a sequence of indeces to wavetable $x(n)$ of length N,

$$z(n) = x((N-1)y(n)),$$

in an operation called *wavetable lookup*.



Considerations for Wavetable Lookup

- Indeces to $x(n)$ are constrained to be *integers* between 0 and $N-1$.
- If signal $y(n)$ is used to index wavetable $x(n)$, its values must be
  - between 0 and $N-1$,
  - **integers**.
- Audio signals in Pd **aren't integers** and don't usually (shouldn't) exceed an **amplitude of 1**.
- Additional processing must be done to signal $y(n)$ to make it usable for wavetable lookup.

Input Except Boundaries

- If $y(n)$ is an input (audio) signal between -1 and 1, it will have to be:
  - **offset**: so that it is positive:
  $$y(n) + 1 \quad \text{(range: 0 to 2)},$$
  - **scaled**: so that it is in range of wavetable size:
  $$(y(n) + 1) \times (N - 1)/2 \quad \text{(range: 0 to } N - 1).$$
- Pd's `phasor~` is between 0 and 1, so only needs to be scaled by $N - 1$.
- If index exceeds bounds $(0, ..., N - 1)$, we may
  1. *clip the input* by substituting 0 or $N - 1$ for any integer that is < 0, or $> N - 1$, respectively.
  2. *wrap the input around* to the end if index < 0, or to the beginning if index $> N - 1$, creating a *circular* wavetable.
- **Problem remains**: values are not integers!

Input is not an Integer

- If input signal $y(n)$ is not an integer, i.e., they fall between two points of the wavetable, we may choose to
  1. *take integer* and truncate fractional part
  2. *round* to nearest integer
  3. *interpolate* between two points of the wavetable.
- Pd's `tabread4~` is an interpolating wavetable reader (an improvement to `tabread~`).

Interpolation (linear)

- Rather than rounding or truncating index values, it is more accurate to **interpolate** $x(n)$.
- **Linear interpolation** of $x(y(n))$:
  – consider a line between neighboring values of $x(n)$ indexed at the floor and ceiling of $y(n)$:
  – a value that would lie on the line is *inferred* depending on the fractional part of the index.
- Example: if $y(n) = 6.5$, the inferred value would be on the line between $x(6)$ and $x(7)$, equidistant from indeces 6 and 7:

$$z(6.5) = \frac{x(6) + x(7)}{2} = .5x(6) + .5x(7)$$

- More generally, for $y(n) = n.\eta$, where n is the integer part and η is the fractional part,

$$z(n + \eta) = (1 - \eta)x(n) + (\eta)x(n + 1),$$

Samplers

- "Sampling" is also used for the process of recording audio into a wavetable then playing it out again.
- A "sample" is also sometimes used (especially commercially) to refer to the the entire wavetable.
- Suppose $x(n)$ is a one-second recording and is of length 44100.
  - if $y(n)$ has a period of 22050 samples, it has a frequency of 2 Hz.
  - the sound will be played back at double the speed.

Sampler Parameters

- Given a sampling rate of f_s and a table length of N samples, the duration of the table is

$$\text{duration} = \frac{N}{f_s} \text{ seconds}$$

- To read the table without changing pitch or length, the period of the phasor is the duration of the table:

$$T_p = \frac{N}{f_s} \text{ seconds}$$

- The corresponding **frequency of the phasor** is

$$f_p = \frac{1}{T_p} = \frac{f_s}{N}$$

- To change sounding frequency by a factor of t:

$$f_p = t\frac{f_s}{N}$$

- h semitones above/below the original frequency is a transposition factor of

$$t = 2^{\pm h/12}$$