Music 170: Quantifying Sound

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Sound as a Wave

- Sound is a **longitudinal** compression wave:

![Waveform Diagram]

- The **waveform** shows alternating regions of low and high pressure, with:
  - **amplitude**: maximum pressure change (Pa).
  - **period**: time to complete one cycle (s).
  - **frequency**: number of cycles per second (Hz).
Wavelength

- Recall, the wavelength is the length of one cycle.

- We can determine the wavelength from the frequency and/or the period, if we have the wave velocity:

\[
\text{wavelength} = \text{period} \times \text{velocity} = \frac{\text{velocity}}{\text{frequency}}
\]
Periods of Audible Frequency Range

- What is the period of the lowest audible frequency?
  - the lowest audible frequency is 20 Hz and has a period of
    \[ \frac{1}{20} = 0.05 \text{ seconds} . \]

- What is the period of the highest audible frequency?
  - the highest frequency is 20,000 Hz and has a period of
    \[ \frac{1}{20000} = 0.00005 \text{ seconds} = 0.05 \text{ milliseconds} . \]
Wavelengths of Audible Frequency Range

- Wavelength and frequency are inversely related.
- Assuming a wave speed of 340 m/s, what is the shortest audible sound wave?
  - the shortest audible wave is the one with the highest frequency:

\[
\text{wavelength} = \frac{\text{velocity}}{\text{frequency}} = \frac{340 \text{ m/s}}{20000 \text{ 1/s}} = 0.017 \text{ m} = 1.7 \text{ cm}.
\]

- the longest audible sound wave?
  - the longest audible wave is the one with the lowest frequency:

\[
\text{wavelength} = \frac{340}{20} \text{ m}, = 17 \text{ m}.
\]
Power and Intensity

• Related to sound pressure are:

1. **sound power** emitted by the source:
   – a fixed quantity, in watts (W),
   – analogous to the wattage rating of a light bulb.

2. **sound intensity** measured some distance from the source:
   – power per unit area carried by wave (W/m$^2$),
   – influenced by interference and environment,
   – analogous to light brightness at different points in a room.
Sound Intensity

- **Sound intensity**, is a measure of the power of a sound that actually contacts an area (e.g. eardrum, microphone, etc).

- The range of human hearing for sound intensity is
  - threshold of audibility \( (I_0) \): \( 10^{-12} \text{ W/m}^2 \)
  - threshold of feeling: \( 1 \text{ W/m}^2 \)

- **Intensity is related to pressure squared**
  - with the actual relationship being
    \[
    I = \frac{p^2}{\rho c},
    \]
    where \( p \) is the pressure, \( \rho \) is the density of air (kg/m\(^3\)), and \( c \) is the speed of sound (m/s).
Linear vs. Logarithmic Scales

- Human hearing is better measured logarithmically.
- On a linear scale,
  - a change between two values is perceived on the basis of their difference
  - e.g., a change from 1 to 2 would be perceived as having the same increase as from 4 to 5.
- On a logarithmic scale,
  - a change between two values is perceived on the basis of their ratio
  - e.g., a change from 1 to 2 would be perceived as having the same increase as a change from 4 to 8.
- Linear: moving one unit to the right adds 1.
  
  ![Linear Scale Diagram]

- Logarithmic: moving right one unit multiplies by 10.
Decibels (dB)

• Sound level is often expressed in decibels (dB).
• To understand decibels, please don’t watch this.
• The Bel,
  – named after telecommunications pioneer Alexander Graham Bell;
  – used to compare two quantities $A$ and $B$:
    \[ \log_{10} \left( \frac{A}{B} \right) = \text{value in Bels} \]
• The decibel is more convenient for expressing relative sound levels (e.g. the power gain of an amplifier):
  – defined as one tenth of a Bel:
    \[ 1 \text{ Bel} = 10 \text{ dB} \]
  – to convert from Bel to dB, multiply by 10.
Comparing Power and Intensity Sources

- The decibel difference between two power levels $\Delta L$ is defined in terms of their power ratio $W_2/W_1$:

$$\Delta L = L_2 - L_1 = 10 \log_{10} \left( \frac{W_2}{W_1} \right) \text{ dB}.$$ 

- Recall, power is proportional to intensity.
- The decibel difference between two signals with intensities $I_1$ and $I_2$ is similarly given by

$$\Delta L = L_2 - L_1 = 10 \log_{10} \left( \frac{I_2}{I_1} \right) \text{ dB}.$$
Power and Intensity Levels

- Decibels are often used as absolute measurements.

- There is an implied fixed reference (e.g. the threshold of audibility).

- **Sound power level** of a source:
  \[
  L_W = 10 \log \left( \frac{W}{W_0} \right) \text{ dB},
  \]
  where \( W_0 = 10^{-12} \text{ W} \).

- **Sound intensity level** at some distance from the source
  \[
  L_I = 10 \log \left( \frac{I}{I_0} \right) \text{ dB},
  \]
  where \( I_0 = 10^{-12} \text{ W/m}^2 \).
Sound Pressure Level

• Recall, intensity is proportional to pressure squared:
\[ I = \frac{p^2}{(\rho c)}. \]

• The sound pressure level \( L_p \) (SPL) is equivalent to sound intensity level in dB:
\[
L_p = 10 \log \frac{I}{I_0} = 10 \log \frac{p^2}{(\rho c I_0)}. 
\]

• The product of \( \rho \) and \( c \) is often approximated by 400:
\[
L_p = 10 \log \frac{p^2}{(\rho c I_0)} = 10 \log \left( \frac{p^2}{4 \times 10^{-10}} \right) \\
= 10 \log \left( \frac{p}{2 \times 10^{-5}} \right)^2 \\
= 20 \log \left( \frac{p}{2 \times 10^{-5}} \right) \\
= 20 \log \left( \frac{p}{p_0} \right) \text{ dB}. 
\]

where \( p_0 = 2 \times 10^{-5} \) is the threshold of hearing for pressure variations.
Increasing distance from a source

- Assuming radiation in free space (and equally in all directions) and a distance $r$ from the source,
  - intensity decreases by $1/r^2$

![Sound intensity diagram]

**Question**: If there is a **doubling** of distance from the source, by what factor will the intensity change?

**Solution**:

- Given an intensity at some initial distance:
  \[ I_1 = \frac{P}{4\pi r^2}, \]

- doubling the distance from the source yields,
  \[ I_2 = \frac{P}{4\pi(2r)^2} = \frac{P}{2^24\pi r^2} = \frac{1}{2^2}I_1, \]

a change in intensity by a factor of $1/2^2$. 
Sound Intensity Level with a Doubling of Distance

• How does the sound level change with a doubling of distance?

  – intensity will drop by a factor of $1/2^2$ or $2^{-2}$ and

  $$L_I = 10 \log \left( \frac{I}{I_0} 2^{-2} \right)$$

  $$= 10 \log \left( \frac{I}{I_0} \right) + 10 \log(2^{-2})$$

  $$= 10 \log \left( \frac{I}{I_0} \right) - 20 \log(2)$$

  $$= 10 \log \left( \frac{I}{I_0} \right) - 20(.3)$$

  $$= 10 \log \left( \frac{I}{I_0} \right) - 6 \text{ dB}.$$  

  – doubling the distance from a source causes a decrease of $6 \text{ dB}$ in the sound level.
SPL with a Doubling of Distance

• We should obtain the same result for pressure as with intensity.

• If intensity decreases by $1/r^2$, then
  – pressure decreases by $1/r$,
  – (intensity is proportional to pressure squared).

• With a doubling of distance, pressure will drop by a factor of $1/2$ or $2^{-1}$,

\[
L_p = 20 \log \left( \frac{p}{p_0} 2^{-1} \right) \\
= 20 \log \left( \frac{p}{p_0} \right) - 20 \log(2) \\
= 20 \log \left( \frac{p}{p_0} \right) - 6 \text{ dB}.
\]
Multiple sources

- When there are multiple sound sources, the total power emitted is the **sum of the power** from each source.

- By how much would the sound level increase when two sources sound simultaneously with equal power?
  - the sound power level would double,

\[
L_W = 10 \log \left( \frac{2W}{W_0} \right)
\]

\[
= 10 \log \left( \frac{W}{W_0} \right) + 10 \log(2)
\]

\[
= 10 \log \left( \frac{W}{W_0} \right) + 3 \text{ dB},
\]

and there would be an increase of 3 dB.

- Similarly, there would be a 3 dB increase in the sound intensity level measured at some distance away from the source.

- This accounts for most cases; the actual result depends on correlation (and interference) of sound sources.